A COMPUTER AIDED DESIGN OF DIGITAL FILTERS

Salih Kayhan Elitas

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A COMPUTER AIDED DESIGN
OF
DIGITAL FILTERS

by

Salih Kayhan Elitas

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Thesis Advisor: S. G. Chan

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The performance of this MTBC filter is compared to that of Butterworth, Chebyshev, transitional Butterworth-Chebyshev filters together with those suggested by other investigators [1]-[3]. It is shown that the stop-band attenuation can be significantly increased without great sacrifice of cut-off slope rate.

Step response of this MTBC filter is also obtained and compared with other filters. Various tabulations as well as graphs of this filter are given for design purposes. A computer program is developed for the design of this filter.

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by

Salih Kayhan ELITAS
Lieutenant,Turkish Navy
B.S.E.E., Naval Postgraduate Scholl,1975
M.S.E.E., Naval Postgraduate School,1976

Submitted in partial fulfillment of the
requirements for the degree of

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Last, but not least, the author wishes to thank his wife Guzin, for her patience and understanding.

I. INTRODUCTION

The digital filter is, as defined by Rabiner et al [10], " a computational process or algorithm by which a digital signal or sequence of numbers (acting as input) is transformed into a second sequence of numbers termed the output digital signal".

The area of digital filtering can be divided into two major subdivisions as Finite Impulse Response (FIR) filters and Infinite Impulse Response (IIR) filters.

During the development of digital signal processing, the interest of the investigators in IIR and FIR filters varied. Before the introduction of the FFT algorithm by Cooley and Tukey (1965) IIR filters were much more efficient than FIR filters. Stocham's work [13] on the FFT method of performing convolution indicated that implementation of high-order FIR filters could be made extremely computationally efficient ; thus, comparison between FIR and IIR filters are no longer strongly biased toward the latter [5]. Because FIR filters require very high orders to produce a sharp attenuation shape, they are not often used for real-time filtering of waveforms. Recently, due to the increase in computing capabilities in digital signal processing and the availibity of long charge transfer device (CTD) tapped delay lines(TDL), FIR filters are favored over IIR filters. However, in applications like design of digital comb filters IIR filters are the unique alternative.

There are three basic design techniques of IIR digital filters [5].

First method is the direct design, which is, appropriately placing poles and zeros to approximate required frequency response.

A second method is to use an optimization procedure to place the poles and zeros to match arbitrary frequency response specifications.

Finally the third technique makes use of highly advanced art of continuous filter design. This technique of designing digital filters from continuous filters by means of mathematical transformations is the most popular IIR digital filter design technique.

Standard Z-transform,Bilinear Z-transform and the matched Z-transform make possible direct transformation from S-domain to Z-domain, preserving essential characteristics of analog frequency response.

Existence of frequency transformations reduces the problem to design a frequency normalized prototype low-pass filter.Then using appropriate frequency transformation, this prototype may be converted into desired band-pass, band-reject or high-pass filter. Popular prototype filters are Butterworth, Chebyshev, elliptic and hybrid transitional filters. Frequency transformations for digital filters are discussed in various literatures ([6] and [7]).

The problem of designing low-pass prototype filters, which possesses better stop-band attenuation and cut-off slope characteristics than existing prototypes has always attracted the researchers in the signal processing area.

Budak and Aronhime suggested [1] modification of maximally flat rational functions by introducing a pair of finite transmission zeros such that the maximally flat

characteristic is maintained but the cut-off slope can be made steeper without great sacrifice of stop-band attenuation.

Dutto Roy [2] investigated a more general case allowing insertion of multiple pairs of transmission zeros, either coincident or distinct.

Introducing multiple pairs of jw-axis zeros in all pole Chebyshev transfer functions are investigated by Agarwal and Sedra [3].

The most attractive feature of these finite zero filters is that they offer the filter designer a great degree of freedom in choosing the location and order of the zeros to trade cut-off slope for stop-band attenuation.

In this thesis, a modified Transitional Butterworth-Chebyshev filter is developed, which is a more general case, introducing finite coincident or distinct multiple pairs of transmission zeros in transitional Butterworth-Chebyshev filter.

Trade-off's between the order of the filter, the order of transmission zeros, stop-band attenuation and cut-off slope are pointed out. Graphs helpful in the design of such filters are obtained.

Performances of Butterworth, Chebyshev, Transitional Butterworth-Chebyshev filters and the designs suggested in references [2] and [3] are compared with those represented in this thesis for the orders of three through eleven. A computer program is developed to implement the filters mentioned above.

In addition, the time-domain response of digital filters

is studied. There are many applications, such as digital MTI filters, for which one is interested in the transient responses of filters that are specified in the frequency domain. Step responses of the filters that are discussed in this report are plotted and compared.

II. DERIVATION OF MODIFIED TRANSITIONAL BUTTERWORTH-CHEBYSHEV FILTERS

A. INTRODUCTION

The most popular technique for designing IIR digital filters is to digitize an analog filter that satisfies the design specifications [5]. There are many techniques for designing analog low-pass prototype filters. Among the well known analog filter classes are the maximally flat (Butterworth) and equal ripple (Chebyshev) filters.

Butterworth filters are simple, excellent in the pass-band and monotonic in both pass-band and stop-band. The Chebyshev filters are superior at and near cut-off frequency and at stop-band.

The transitional Butterworth-Chebyshev (TBC) filters combine the desirable attributes of these two filters in a single approximation that is given by

$$|F(j\omega)|^2 = \frac{1}{1+\varepsilon^2 \omega^{2k} C_{n-k}^2(\omega)} \qquad (2\text{-}1)$$

where

k= Weighting factor

n= Order of filter

$C_{n-k}(w)$= $(n-k)^{th}$ order chebyshev polynomial.

When k=n, $|F(jw)|^2$ is identical with the Butterworth function. When k=0, $|F(jw)|^2$ is identical with the Chebyshev

function. With a varying value of k, a TBC filter possesses some characteristics of each. As k approaches to n, a TBC filter behaves more like a Butterworth filter, as k approaches to zero, it behaves more like a Chebyshev filter.

In this chapter, modification of TBC filters by introducing finite coincident or distinct multiple pairs of transmission zeros will be discussed. It will be shown that, using a weighting factor k, and the location and order of inserted zeros as parameters, attenuation in the stop-band may be traded for sharpness of the cut-off characteristics.

Expressions for the cut-off slope and minimum attenuation in the stop-band are derived in terms of order of the filter, weighting factor, location, and order of inserted zeros.

B. MODIFICATION OF THE TBC FILTERS WITH COINCIDENT TRANSMISSION ZEROS

Introducing m identical pairs of transmission zeros at $\pm j w_o$ to eq. (2-1), we have

$$|F(j\omega)|^2 = \frac{(\omega_o^2-\omega^2)^{2m}}{(\omega_o^2-\omega^2)^{2m} + K\,\varepsilon^2\,\omega^{2k}\,C_{n-k}^2(\omega)} \qquad (2\text{-}2)$$

In order to normalize $|F(jw)|^2$, i.e. to force $|F(jw)|^2$ to be equal to 1/2 at $w=1$, the constant K should be

$$K = \frac{(\omega_o^2-1)^{2m}}{\varepsilon^2} \qquad (2\text{-}3)$$

Then eq. (2-1) becomes

$$|F(j\omega)|^2 = \frac{(\omega_0^2-\omega^2)^{2m}}{(\omega_0^2-\omega^2)^{2m} + (\omega_0^2-1)^{2m}\,\omega^{2k}\,C_{n-k}^2(\omega)} \tag{2-4}$$

where n>2m, because of the low-pass characteristics of the function, and the Chebyshev polynomial C_n^2 (w) is defined by

$$C_n^2(\omega) = \begin{cases} \cos^2(n\cos^{-1}\omega) & ,\ \omega \leq 1 \quad (2\text{-}5) \\ \cosh^2(n\cosh^{-1}\omega) \cong 2^{2n-2}\,\omega^{2n} & ,\ \omega > 1 \quad (2\text{-}6) \end{cases}$$

1. Slope at cut-off frequency

Substituting eq. (2-5) into eq. (2-4), we obtain

$$|F(j\omega)|^2 = \frac{(\omega_0^2-\omega^2)^{2m}}{(\omega_0^2-\omega^2)^{2m} + (\omega_0^2-1)^{2m}\,\omega^{2k}\,\cos^2[(n-k)\cos^{-1}\omega]} \tag{2-7}$$

Let

$$f(\omega) = (\omega_0^2-\omega^2)^{2m} \tag{2-8}$$

$$h(\omega) = \omega^{2k}\cos^2[(n-k)\cos^{-1}\omega] \tag{2-9}$$

Using these values, eq. (2-7) becomes

$$|F(j\omega)|^2 = \frac{f(\omega)}{f(\omega) + (\omega_0^2-1)^{2m}\,h(\omega)} \tag{2-10}$$

Taking the derivative of eq. (2-10) with respect to w, we get

$$|F(j\omega)|' = \frac{(\omega_o^2-1)^{2m}\left[f'(\omega)\,h(\omega) - f(\omega)\,h'(\omega)\right]}{2\,|F(j\omega)|\left[f(\omega) + (\omega_o^2-1)^{2m}\,h(\omega)\right]^2} \qquad (2\text{-}11)$$

At the cut-off frequency, w=1, we have

$$f(\omega) = (\omega_o^2-1)^{2m} \qquad (2\text{-}12)$$

$$f'(\omega) = -4m\,(\omega_o^2-1)^{2m-1} \qquad (2\text{-}13)$$

$$h(\omega) = 1 \qquad (2\text{-}14)$$

$$h'(\omega) = 2k + 2(n-2)^2 \qquad (2\text{-}15)$$

$$|F(j\omega)| = 1/\sqrt{2} \qquad (2\text{-}16)$$

Substituting these values into eq. (2-11) and simplifiying, we obtain

$$|F(j\omega)|' = \left[\frac{m}{\sqrt{2}\,(\omega_o^2-1)} + \frac{k + (n-k)^2}{2\sqrt{2}}\right] \qquad (2\text{-}17)$$

For k=n, the result agrees with the cut-off slope of MB function, which is derived in [2]. For k=0, the result agrees with the cut-off slope of MC function as derived in [3].

2. Stop-band characteristics

In general, stop-band characteristics of finite zero filters will be of the form shown in FIG.1*.

* Because of the inherent limitations in the plotting subroutine utilized to provide the graphs in this thesis, it was necessary to add supplementary axes to show proper scaling for some of the graphs.

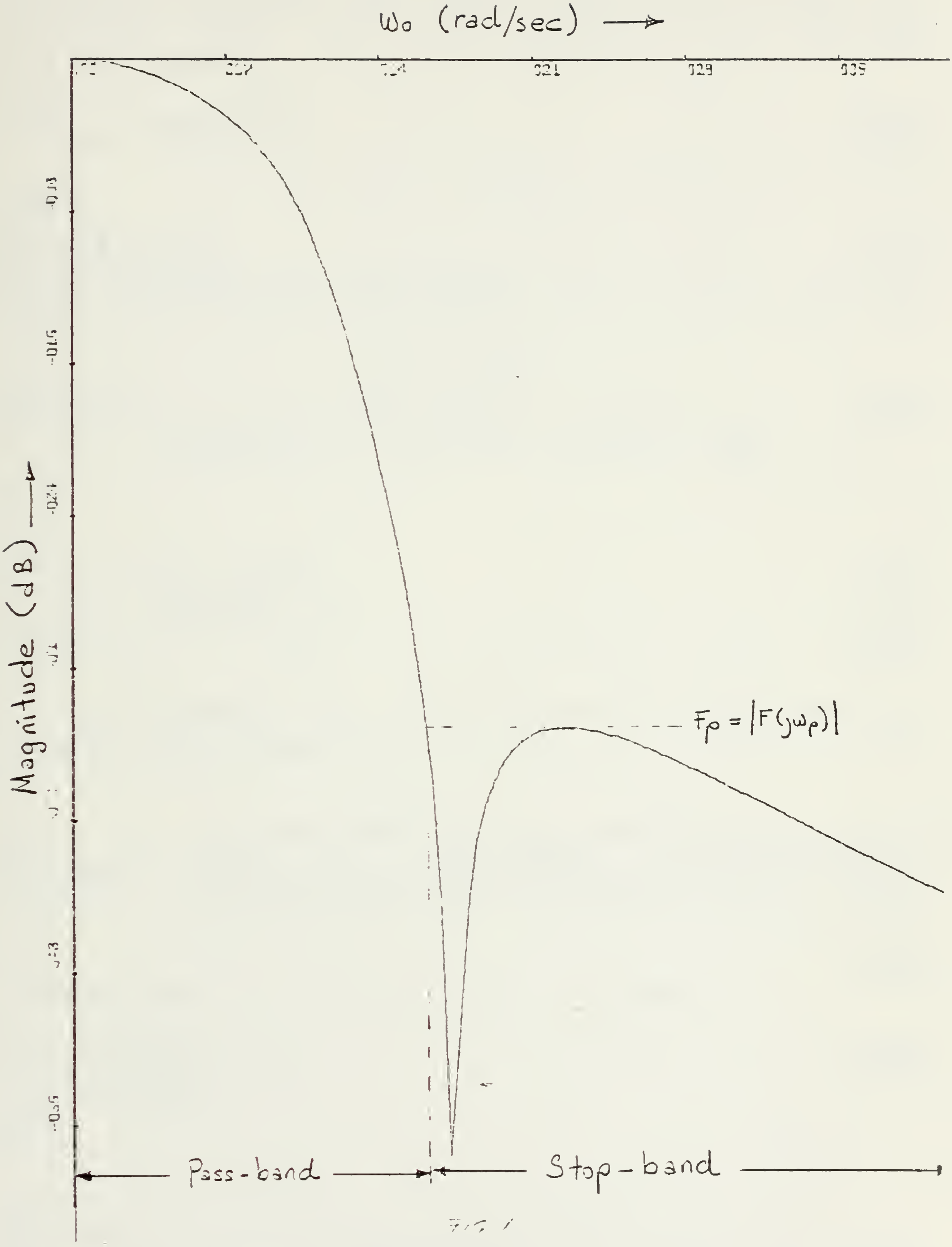

Figure 1 - GENERAL STOP-BAND CHARACTERISTICS OF THE FINITE ZERO FILTERS

The zeros introduced will cause a peak in the stop-band, at a frequency $w_p > w_0$. The minumum attenuation in the stop-band may be defined as

$$\alpha_{MTBC} = 20 \log F_p \tag{2-18}$$

where

$$F_p = \left| F(j\omega_p) \right| \tag{2-19}$$

In the stop-band eq. (2-4) becomes

$$\left| F(j\omega) \right|^2 = \frac{(\omega_0^2 - \omega_p^2)^{2m}}{(\omega_0^2 - \omega_p^2)^{2m} + (\omega_0^2 - 1)^{2m} \; 2^{2(n-k)-2} \; \omega_p^{2n}} \tag{2-20}$$

Let

$$f(\omega_p) = (\omega_0^2 - \omega_p^2)^{2m} \tag{2-21}$$

$$h(\omega_p) = 2^{2(n-k)-2} \; \omega_p^{2n} \tag{2-22}$$

Taking derivatives of these values and substituting them into eq. (2-11) results in

$$\left| F(j\omega_p) \right|' = \frac{(\omega_0^2 - 1)^{2m} \; 2^{2(n-k)-2} \; \omega^{2n-1} (\omega_0^2 - \omega_p^2)^{2m-1} \left[-4m\,\omega_p^2 - (\omega_0^2 - \omega_p^2)\, 2n \right]}{2 \left| F(j\omega_p) \right| \left[(\omega_0^2 - \omega_p^2)^{2m} + (\omega_0^2 - 1)^{2m} \; 2^{2(n-k)-2} \; \omega_p^{2n} \right]^2} \tag{2-23}$$

Combining eq. (2-19) and eq. (2-23), we obtain

$$\omega_p = \sqrt{\frac{n}{n-2m}} \; \omega_0 \tag{2-24}$$

Thus

$$F_p = \frac{\quad}{1 + (\omega_0^2 - 1)^{2m} \; 2^{2(n-m-k-1)} \; \dfrac{n^n}{(n-2m)^{n-2m} \; m^{2m}} \; \omega_0^{2(n-2m)}} \tag{2-25}$$

And minumum stop-band attenuation will be given by

$$\alpha_{MTBC} = 10\, n \log\left(\frac{n}{n-2m}\right) + 20\, m \log(\omega_o) + 20\, m \log\left[\frac{n-2m}{n} \cdot \frac{\omega_o^2 - 1}{\omega_o^2}\right] + 6(n-m-k-1) \quad (2\text{-}26)$$

Plots of stop-band attenuation and cut-off slope of MTBC filters with two coincident transmission zeros, for orders 3 through 11 are given in figures 2 and 3.

C. MODIFICATION OF TBC FILTERS WITH DISTINCT TRANSMISSION ZEROS

Consider the n^{th} order TBC function with m pairs of finite distinct zeros at $\pm j w_i$, where i=1,2,...,m and $w_i > 1$.

The magnitude squared function of TBC function with distinct transmission zeros will be given by

$$|F(j\omega)|^2 = \frac{\prod_{i=1}^{m} (\omega_i^2 - \omega^2)^2}{\prod_{i=1}^{m} (\omega_i^2 - \omega^2)^2 + \prod_{i=1}^{m} (\omega_i^2 - 1)^2 \, \omega^{2k} \, C_{n-k}^2(\omega)} \quad (2.27)$$

Let

$$g(\omega) = \prod_{i=1}^{m} (\omega_i^2 - 1)^2$$

$$h(\omega) = \prod_{i=1}^{m} (\omega_i^2 - \omega^2)^2$$

Putting these values into eq. (2-27) results in

$$|F(j\omega)|^2 = \frac{1}{1 + \dfrac{g(\omega)\, \omega^{2k} C_{n-k}^2(\omega)}{h(\omega)}} \quad (2.28)$$

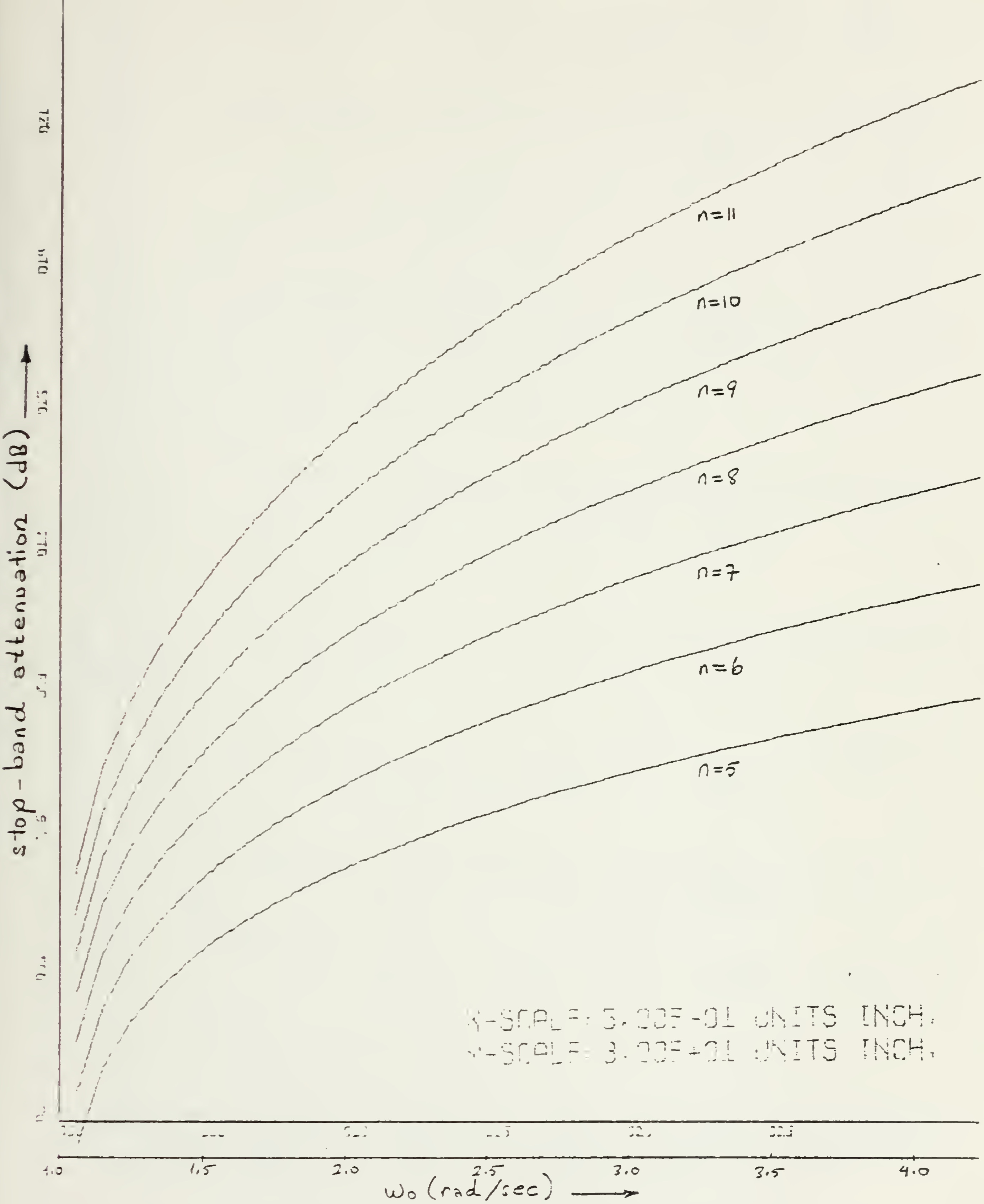

Figure 2 - STOP-BAND ATTENUATION OF MTBC FILTER WITH TWO COINCIDENT ZEROS

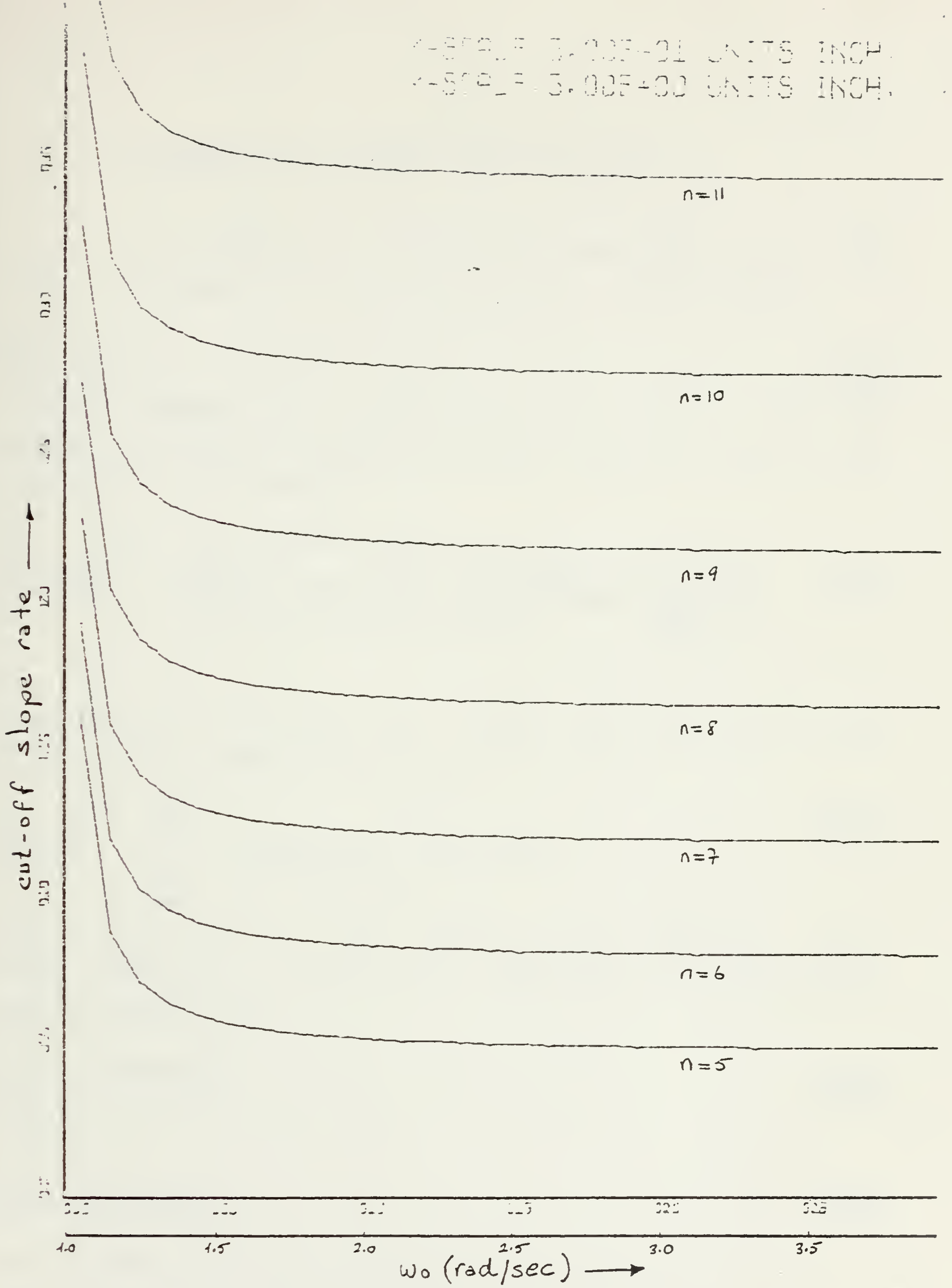

Figure 3 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO COINCIDENT ZEROS

1. Stop-band and cut-off characteristics

The frequency w_p at which the stop-band peak will occur, is given by

$$\left|F(j\omega)\right|'\Big|_{\omega=\omega_p} = 0 \tag{2.29}$$

Using eq. (2-6) and taking derivative of eq. (2-28) with respect to w, we obtain

$$\left|F(j\omega)\right|' = \frac{g(\omega)\, 2^{2(n-k)-2}\, \omega^{2n-1}\left[2n\, h(\omega) - \omega\, h'(\omega)\right]}{2\left|F(j\omega)\right| h^2(\omega)\left[1 + g(\omega)\, 2^{2(n-k)-2}\, \frac{\omega^{2n}}{h(\omega)}\right]^2} \tag{2.30}$$

Combining eq. (2-29) and eq. (2-30) the equation to be solved for w_p may be found to be

$$2\,\omega_p^2\left[\sum_{i=1}^{m} \frac{1}{(\omega_i^2-\omega_p^2)^2}\right] + \Omega = 0 \tag{2.31}$$

which agrees with eq. (2-24) when w_i's are identical. Using real solutions of eq. (2-31), stop-band attenuation peaks may be found to be

$$\alpha_i = 20 \log F_{pi} \tag{2.32}$$

where

$$F_{pi} = \left|F(j\omega_{pi})\right| \tag{2.33}$$

and minimum stop-band atttenuation is given by

$$\alpha_{min} = \max\left\{\alpha_i\right\} \tag{2.34}$$

Using eq. (2-5) and differentiating eq. (2-27) with respect to w, the cut-off slope becomes

$$|F(j\omega)|' = -\frac{1}{2\sqrt{2}}\left[k + (n-k)^2 + 2\sum_{i=1}^{m}\frac{1}{\omega_i^2 - 1}\right] \qquad (2.35)$$

which agrees with eq. (2-17) when the w_i's are identical.

Plots of cut-off slope and stop-band attenuation of MTBC filters with two distinct transmission zeros, for orders 3 through 11 are given in figures 3-17.

D. SUMMARY

The Transitional Butterworth-Chebyshev filter combine the best features of the Butterworth and Chebyshev filters. A Modified Transitional Butterworth-Chebyshev filter, obtained by introducing finite transmission zeros to a Transitional Butterworth-Chebyshev filter, possesses cut-off slope and stop-band attenuation, which are dependent on the modification parameters w_0 and m. The closer the w_0 is to unity, the steeper the cut-off slope. However, this improvement in the cut-off region results in degradation of stop-band attenuation. Thus, using w_0 and m as parameters, an advantageous trade between attenuation in stop-band and sharpness of the cut-off characteristics can be made. Graphs are given to serve as guides in trading cut-off slope for stop-band attenuation.

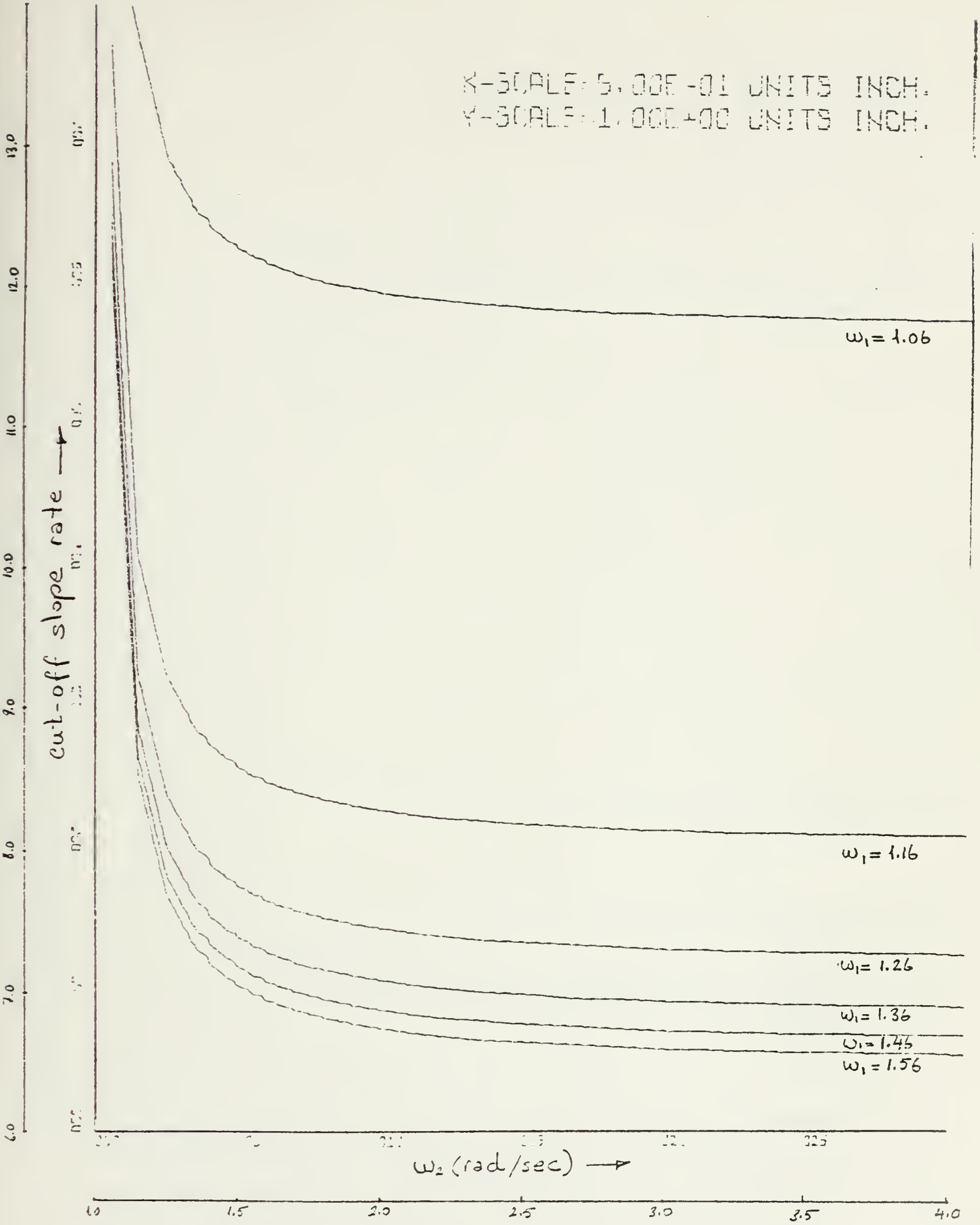

Figure 4 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO DISTINCT ZEROS (w=1•06, n=5)

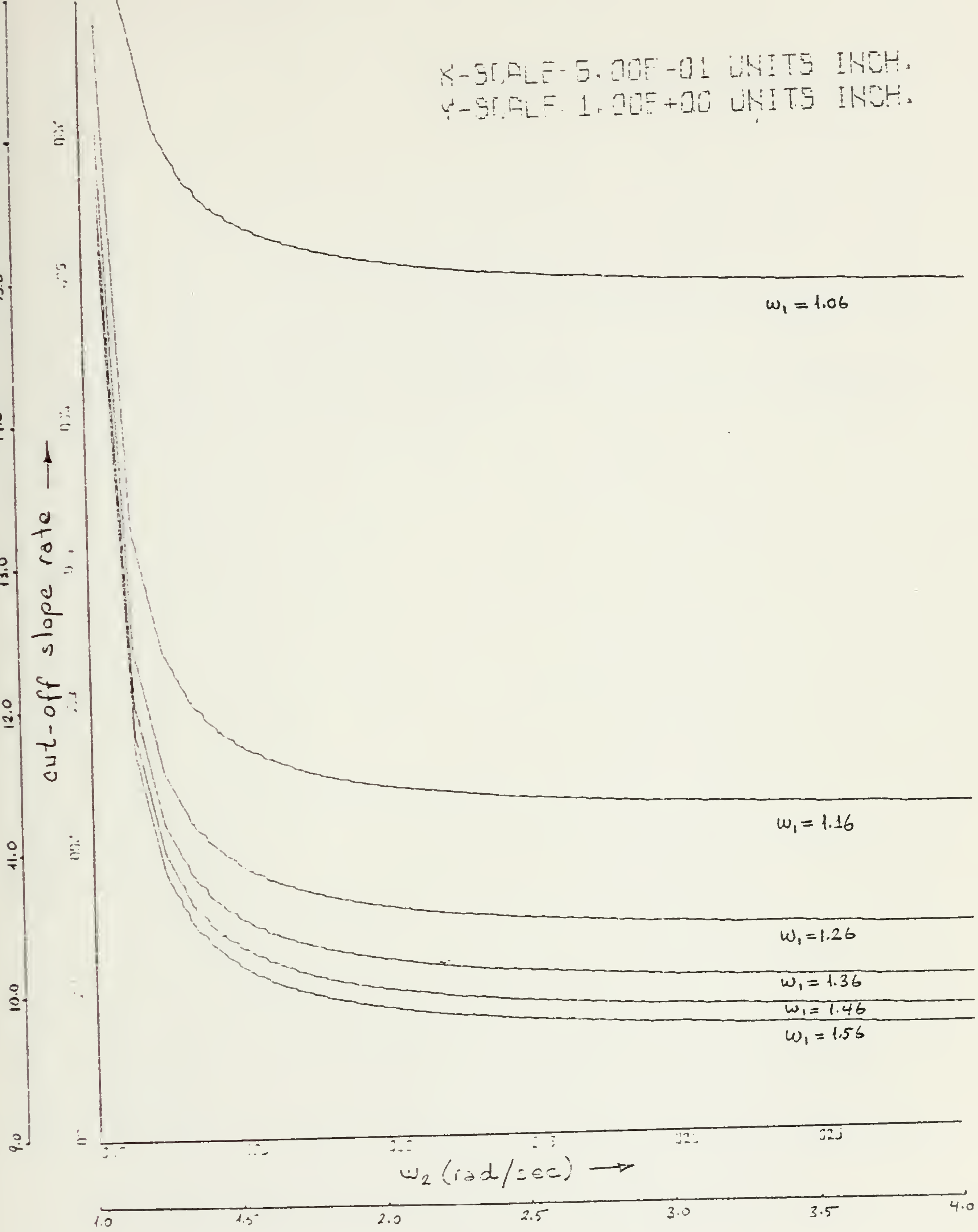

Figure 5 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO DISTINCT ZEROS ($w = 1{\cdot}06$, $n=6$)

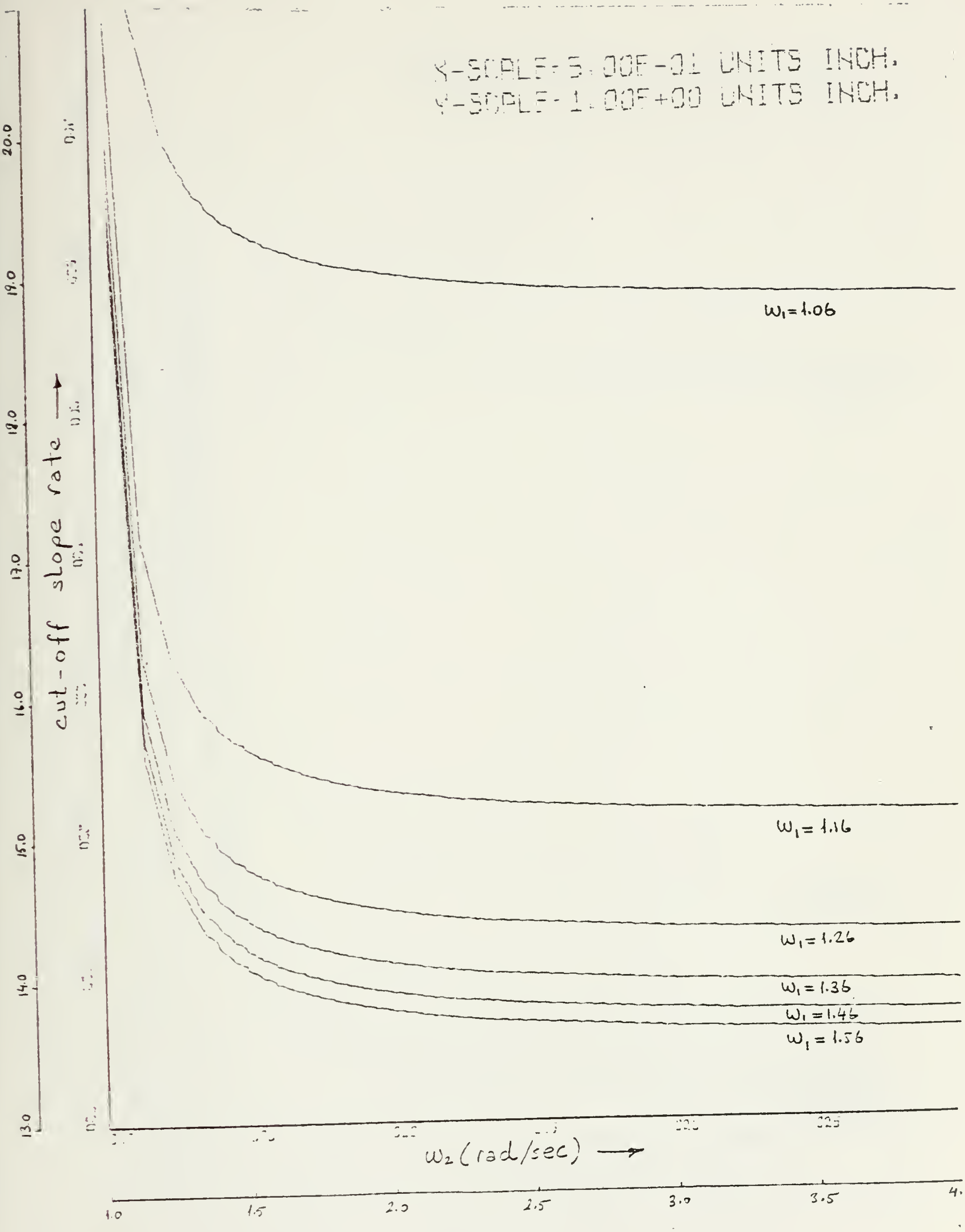

Figure 6 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO DISTINCT ZEROS (w =1•06, n=7)

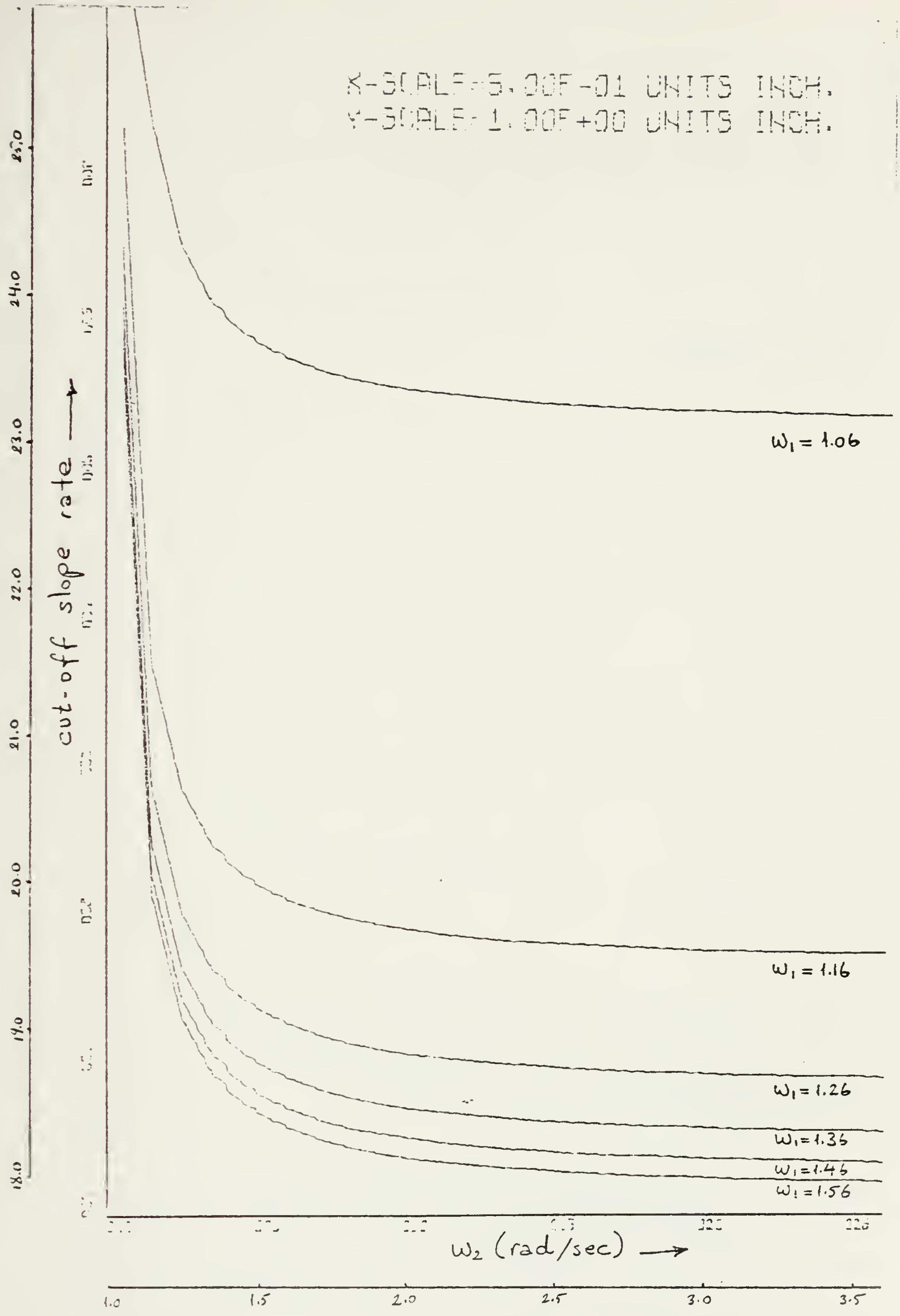

Figure 7 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO DISTINCT ZEROS (w =1•06), n=8)

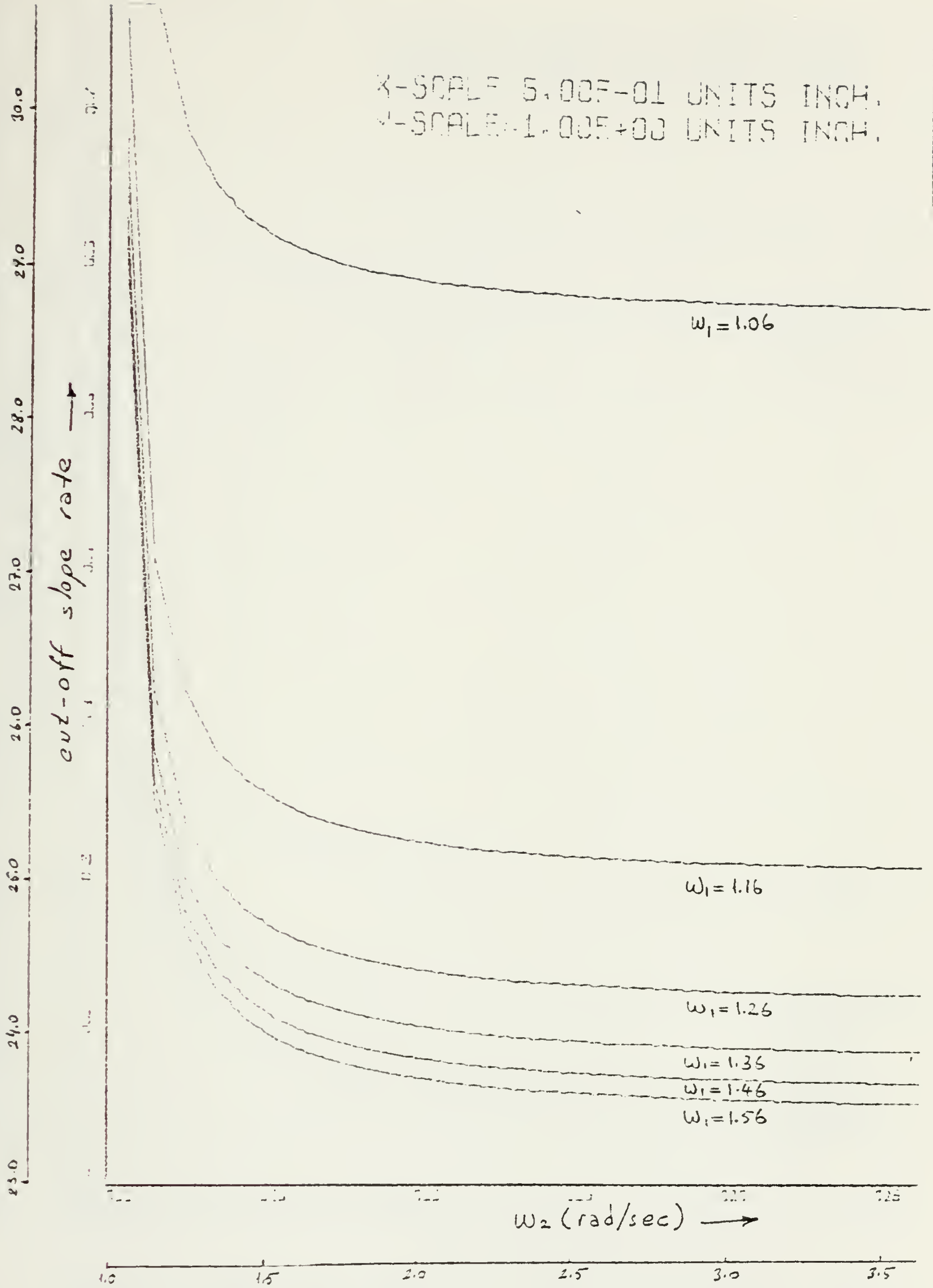

Figure 8 - CUT-OFF SLOPE OF MTBC FILTER WITH TWO DISTINCT ZEROS (w =1•06, n=9)

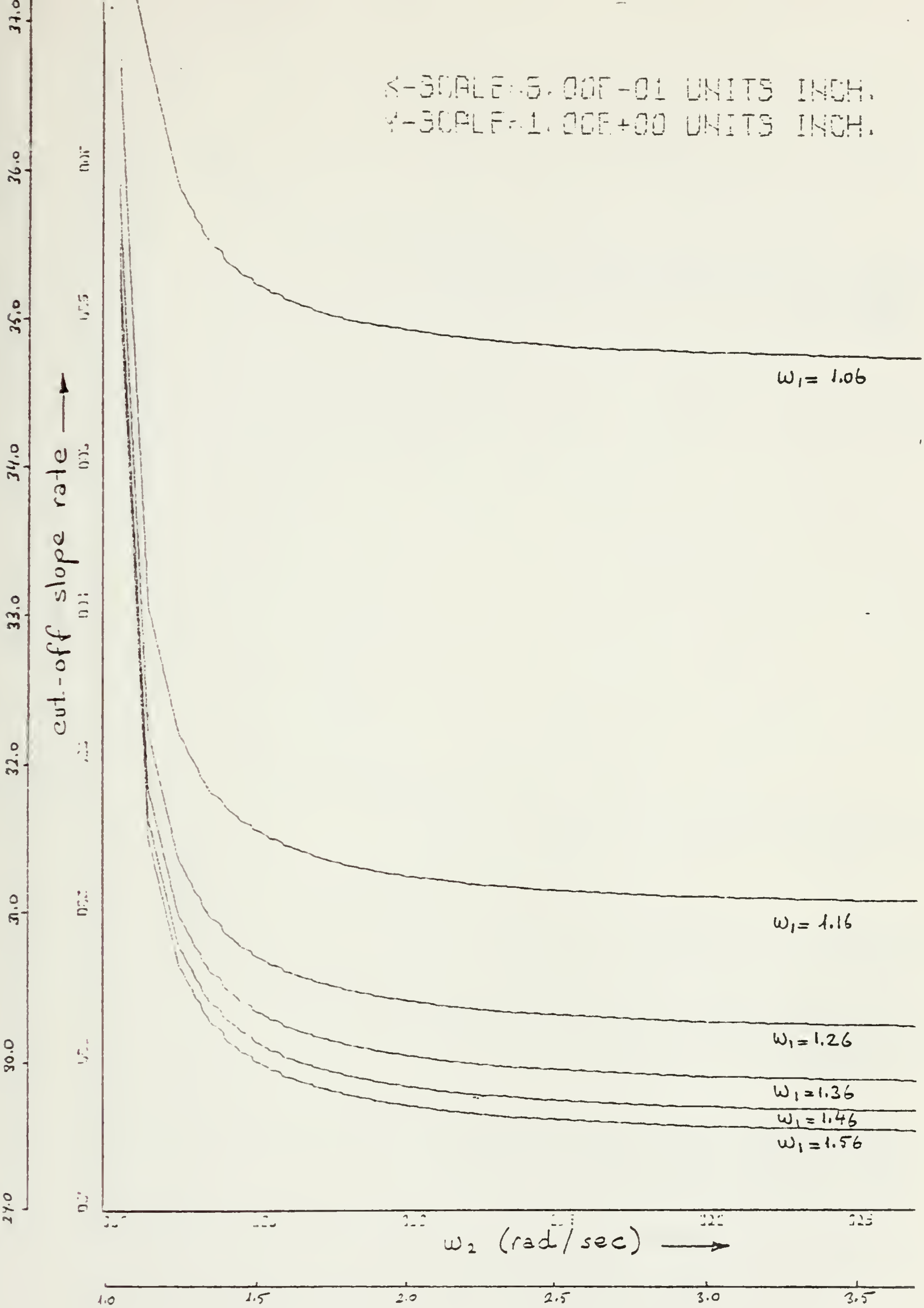

Figure 9 - CUT-OFF SLOPE OF MTBC FILTER WITH DISTINCT TRANSMISSION ZEROS (w =1•06 ,n=10)

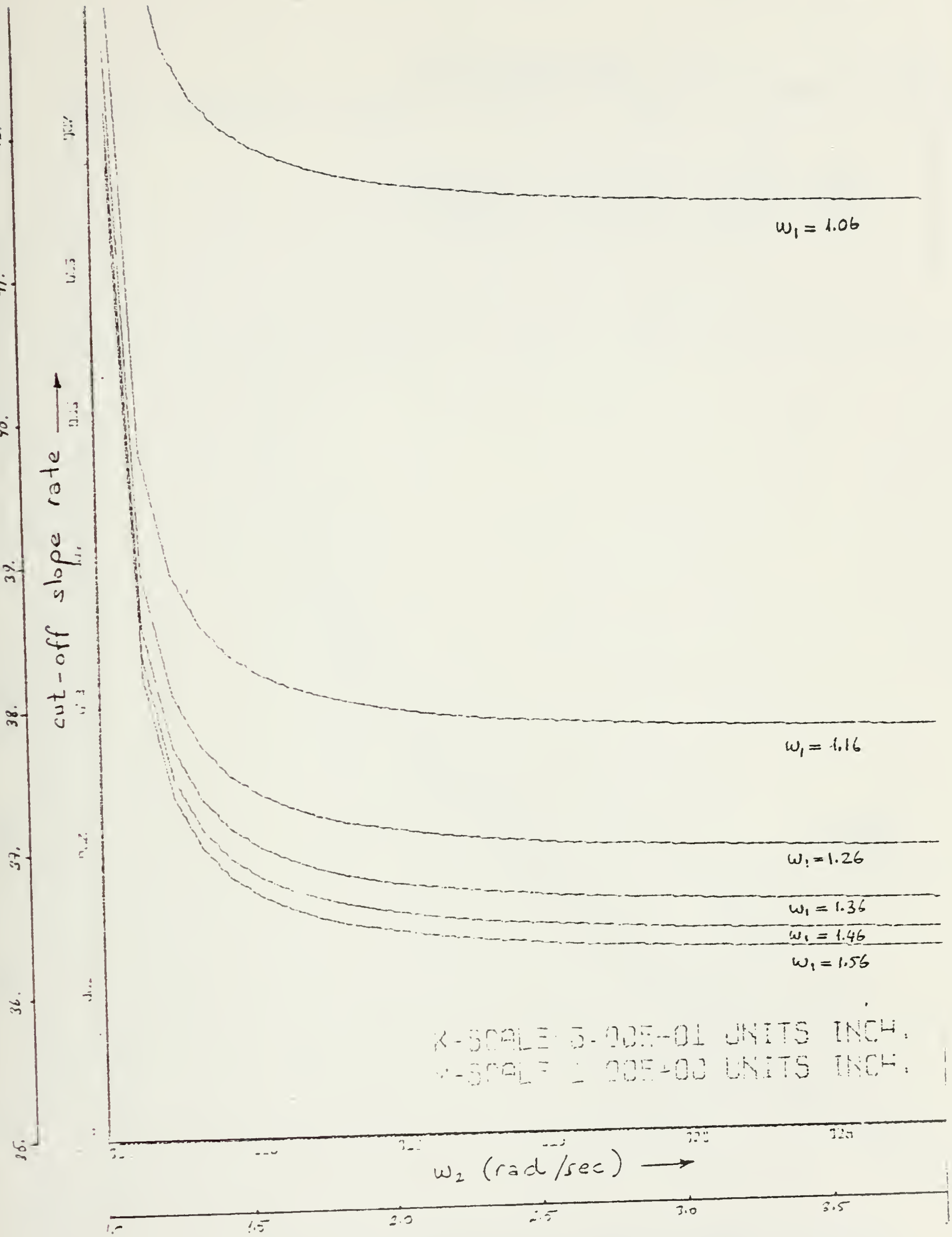

Figure 10 - CUT-OFF SLOPE OF MTBC FILTER WITH DISTINCT TRANSMISSION ZEROS (w =1•06 ,n=11)

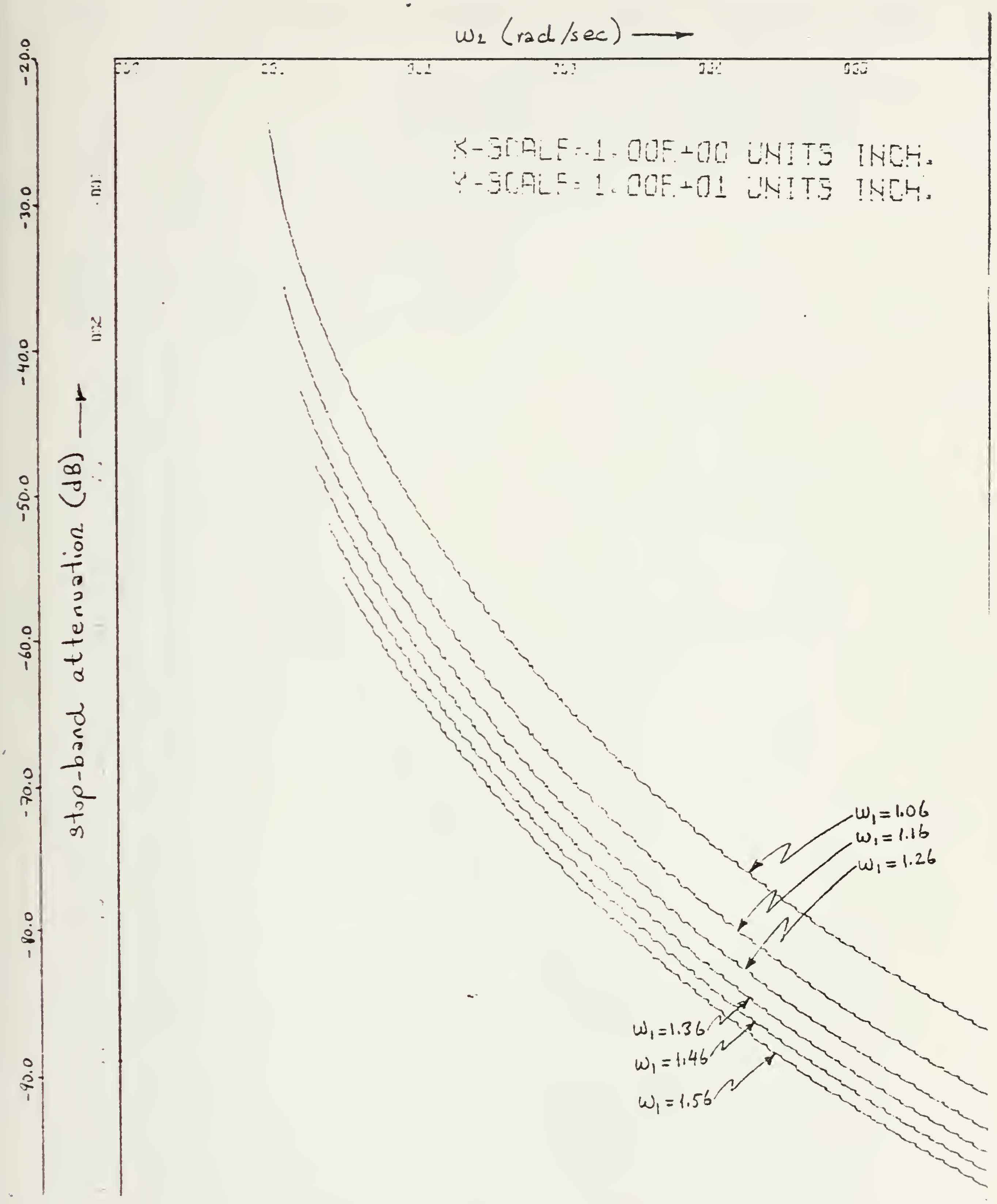

Figure 11 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS (w =1•06, n=5)

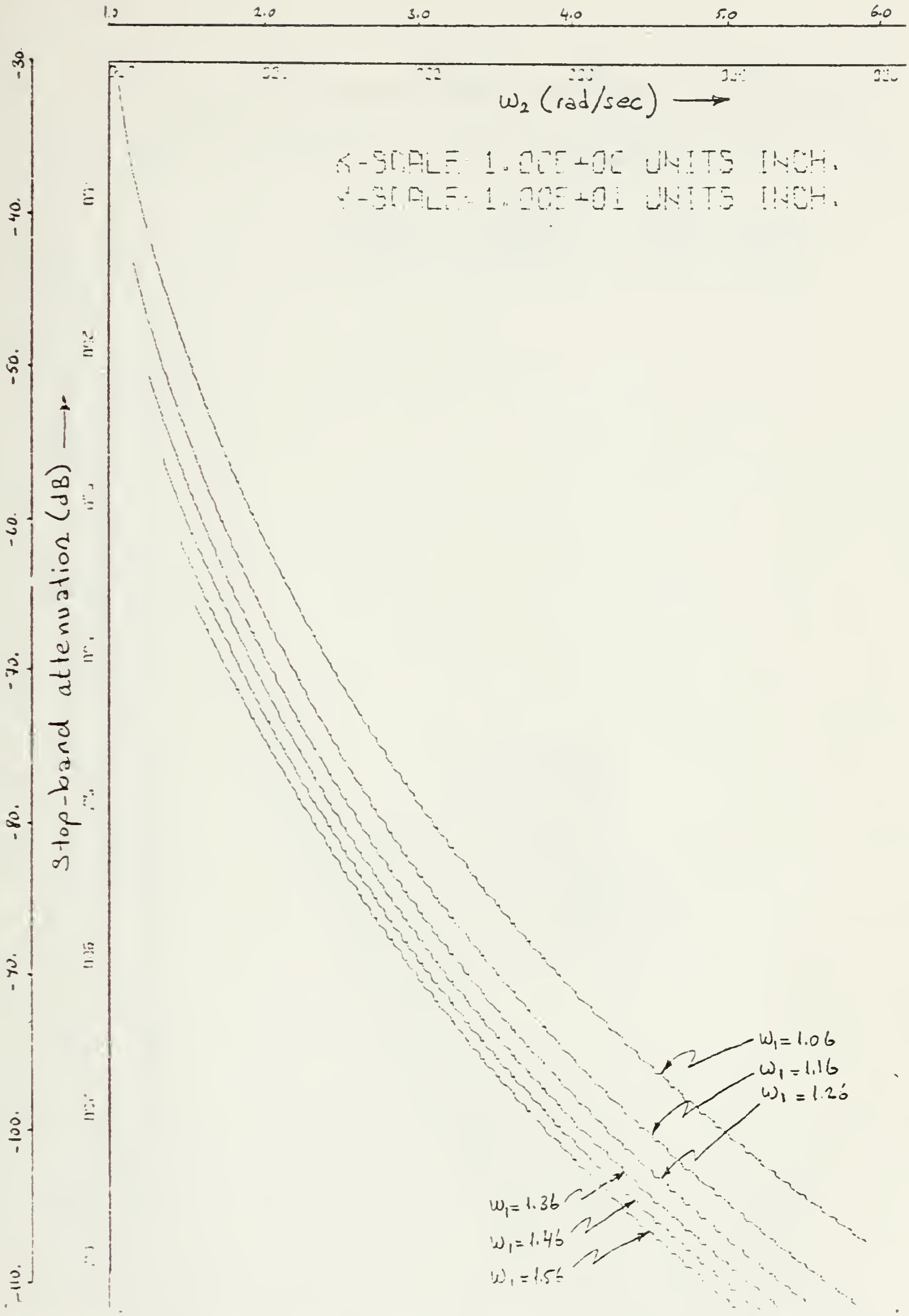

Figure 12 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS ($w = 1.06$, $n=6$)

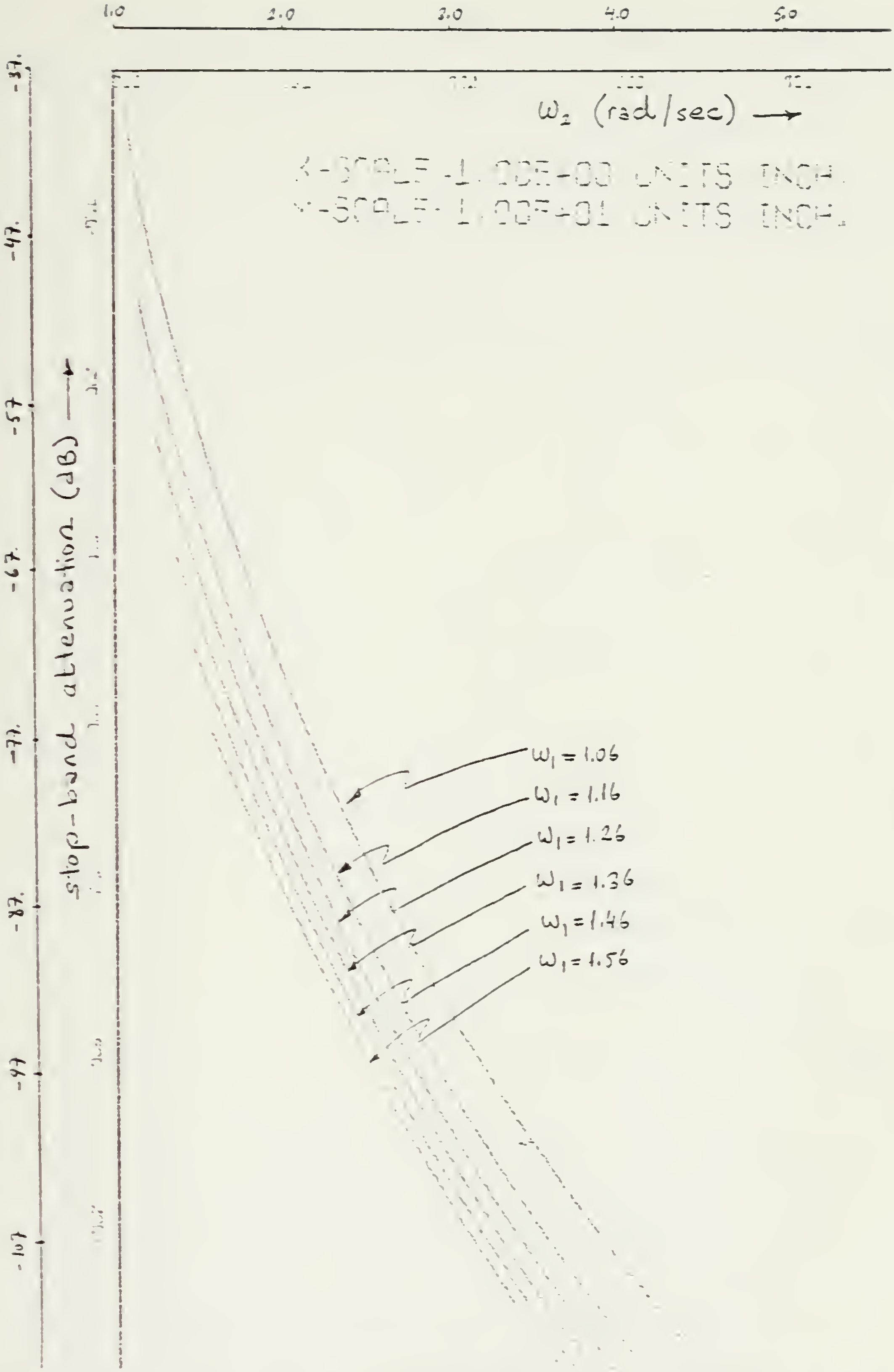

Figure 13 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS (w =1•06, n=7)

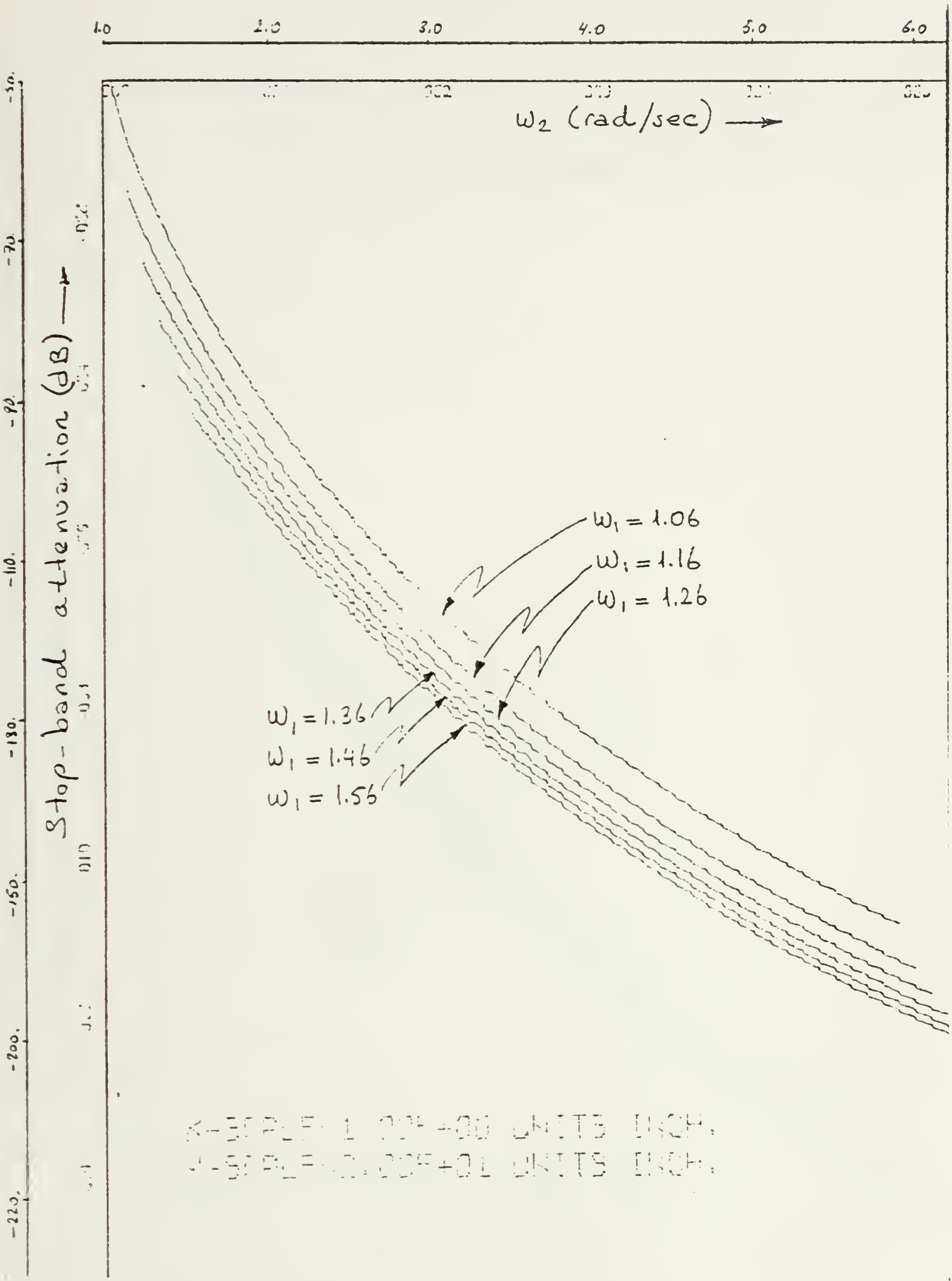

Figure 14 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS (w =1•06, n=8)

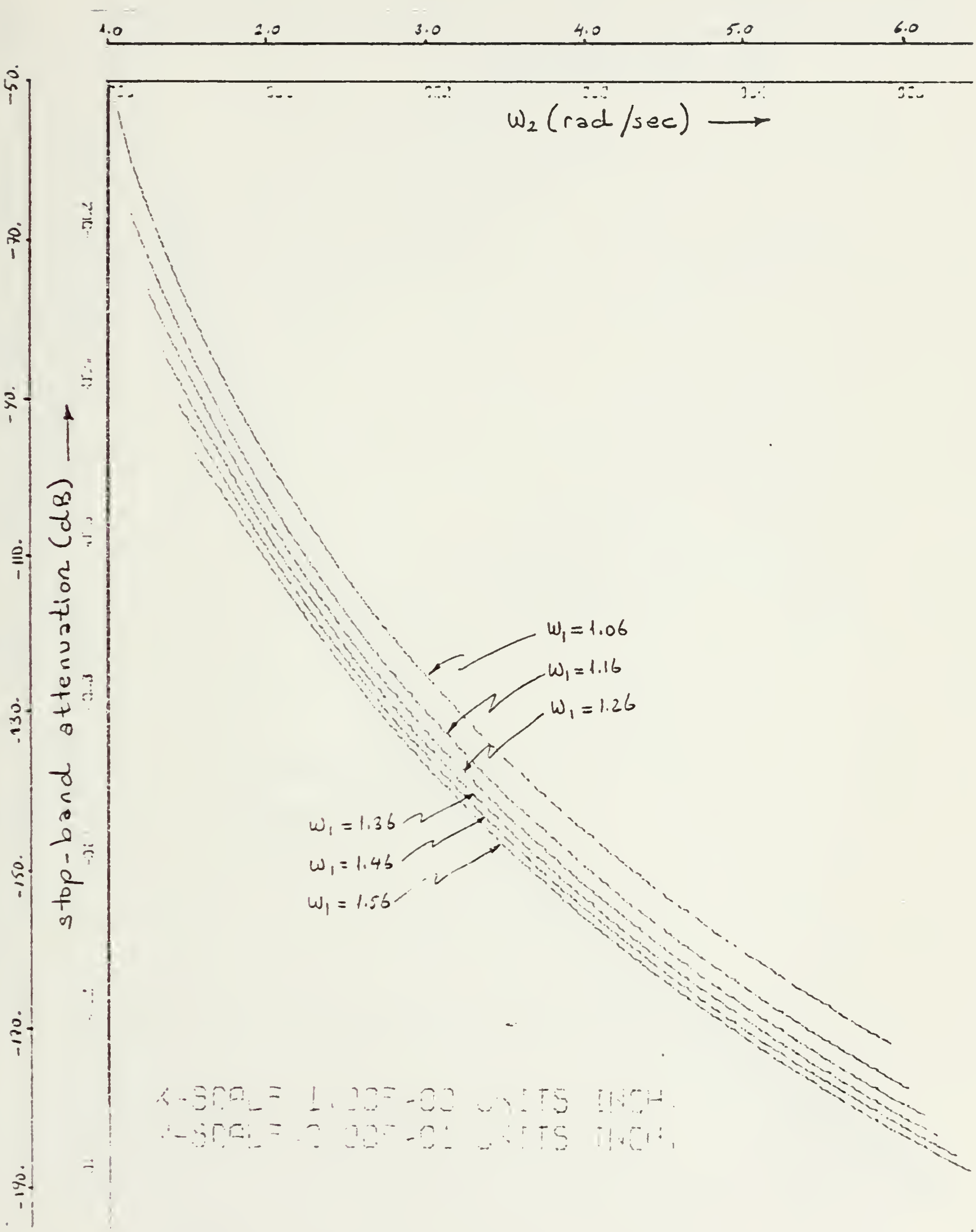

Figure 15 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS (w =1•06, n=9)

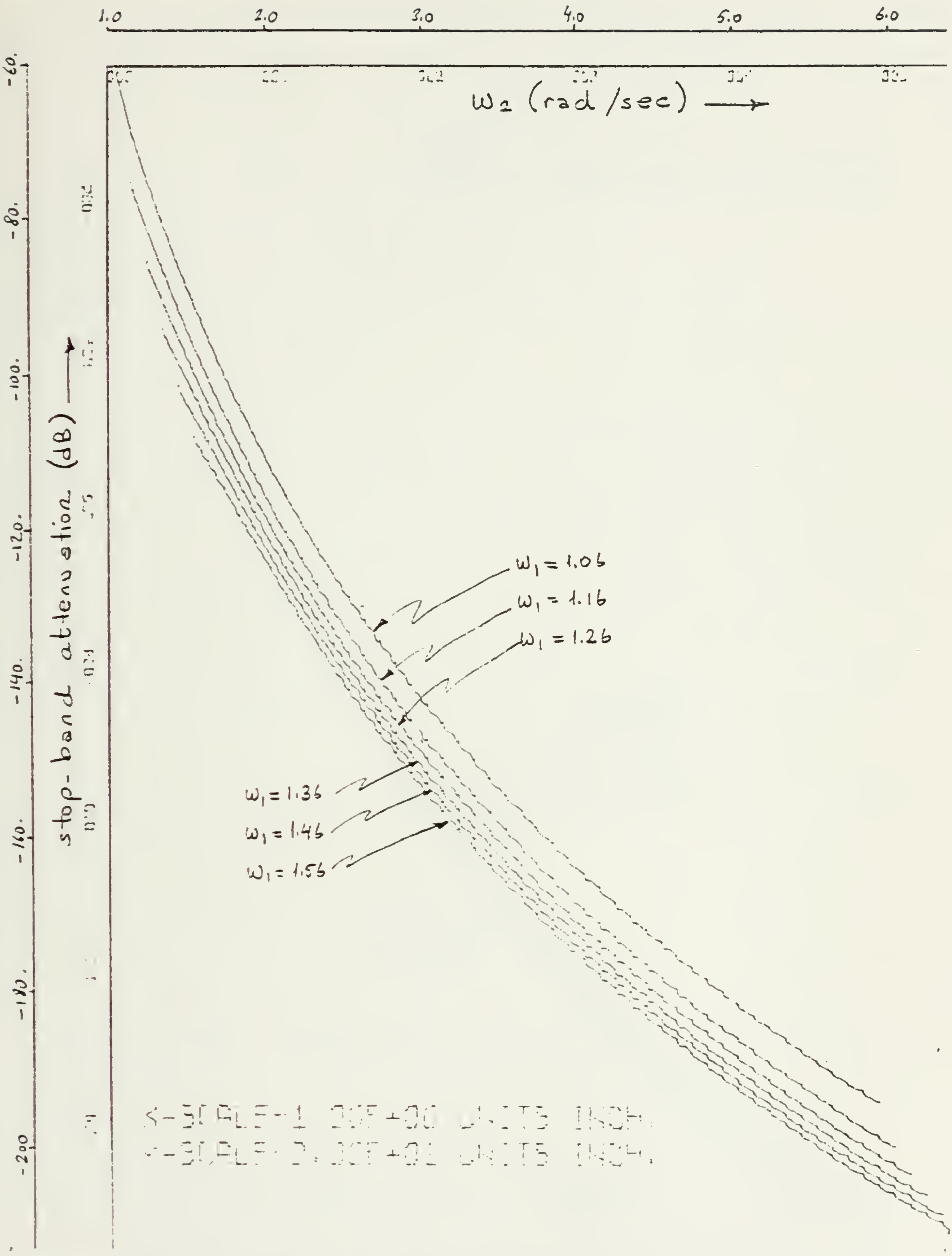

Figure 16 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS ($w = 1 \bullet 06$, $n=10$)

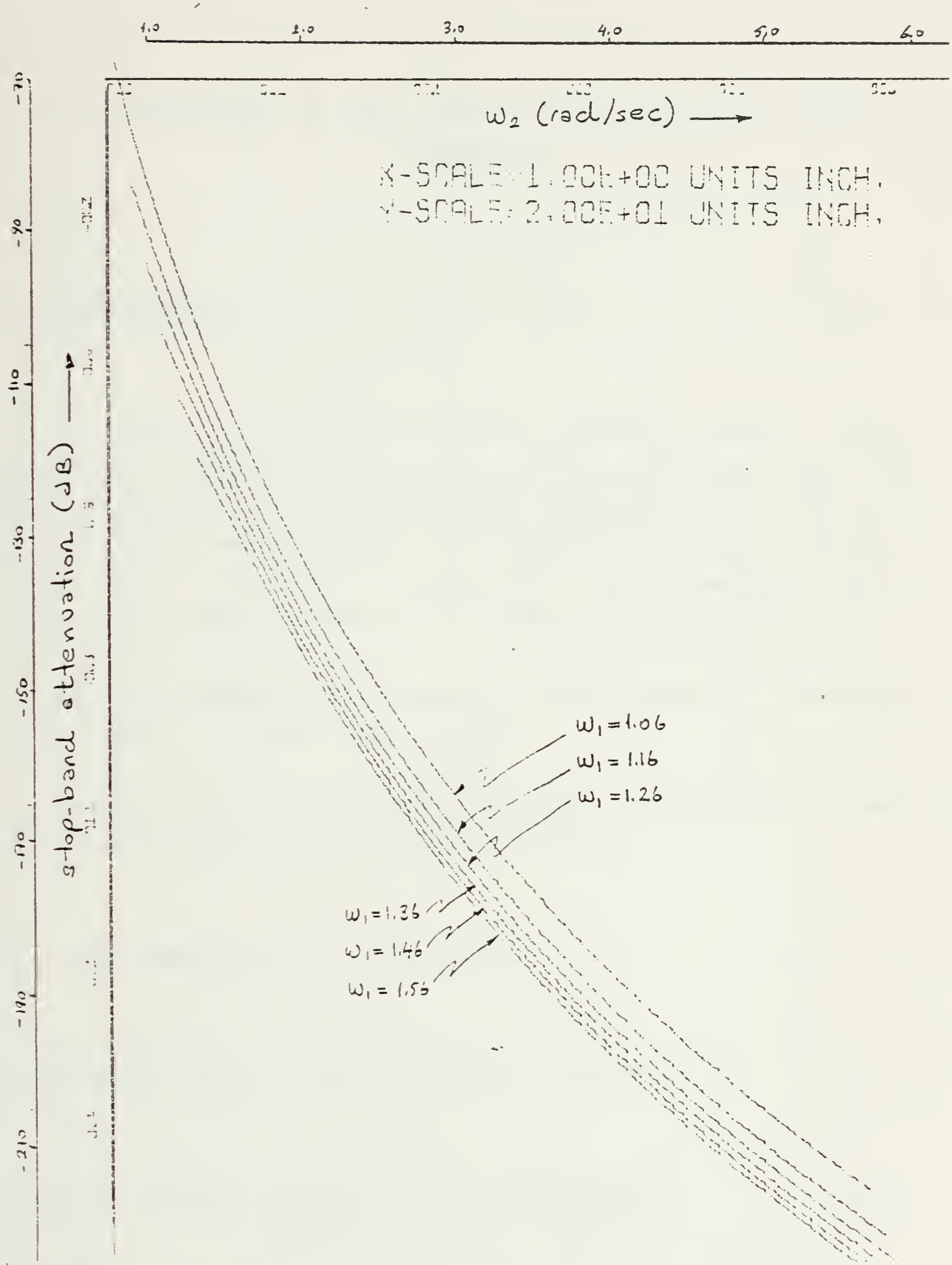

Figure 17 - STOP-BAND ATTENUATION OF MTBC FILTERS WITH TWO DISTINCT TRANSMISSION ZEROS (w =1•06, n=11)

III. COMPARISON OF MTBC FUNCTION WITH B,MB,C,MC AND TBC FUNCTIONS

A. INTRODUCTION

In chapter 2, derivation of MTBC filter is given. It is shown that, stop-band attenuation and cut-off slope rate of this filter depend on modification parameters m, k, and w_o. So the filter designer have the flexibility to trade cut-off slope for stop-band attenuation by changing these parameters, without changing the order of the filter.

In this chapter, performance of MTBC filter is compared with B, MB, C, MC, and TBC filters.

Formulas for stop-band attenuations and cut-off slopes of all these filters are given in Table I.

B. MTBC FUNCTION V.S. B FUNCTION

Cut-off slope of MTBC function is given by

$$\gamma_{MTBC} = n\,\gamma_B + \frac{m}{\sqrt{2}\,(\omega_0^2-1)} + \frac{k(k+1-2n)}{2\sqrt{2}} \qquad (3\text{-}1)$$

ATTENUATIONS

TYPE OF APPROXIMATION	SLOPE AT CUT-OFF	STOP-BAND ATTENUATION
BUTTERWORTH (B)	$\gamma = \frac{n}{2\sqrt{2}}$	$\alpha_B = 20 \times n \times \log \omega$
CHEBYSHEV (C)	$\gamma_C = \frac{n^2}{2\sqrt{2}}$ $= \gamma_B \times n$	$\alpha_C = 20 \times n \times \log \omega + 6(n-1)$ $= \alpha_B + 6(n-1)$
TRANSITIONAL BUTTERWORTH-CHEBYSHEV (TBC)	$\gamma_{TBC} = \frac{n^2}{2\sqrt{2}} + \frac{k(k+1-2n)}{2\sqrt{2}}$ $= n\gamma_B + \frac{k(k+1-2n)}{2\sqrt{2}}$ $= \gamma_C + \frac{k(k+1-2n)}{2\sqrt{2}}$	$\alpha_{TBC} = 20n \log \omega + 6(n-k-1)$ $= \alpha_B + 6(n-k-1)$ $= \alpha_C - 6k$

TABLE I• CONTINUED

TYPE OF APPROXIMATION	SLOPE RATE AT CUT-OFF	STOP-BAND ATTENUATION
MODIFIED BUTTERWORTH	$\gamma_{MB} = \gamma_B + \frac{m}{(\omega_o^2-1)}$	$\alpha_{MB} = \alpha_B - 6m + 20m \log\left[\frac{n-2m}{m} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right]$
MODIFIED CHEBYSHEV	$\gamma_{MC} = \gamma_C + \frac{m}{(\omega_o^2-1)}$	$\alpha_{MC} = \alpha_B + 6(n-m-1) + 20m \log\left[\frac{n-2m}{m} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right]$
MODIFIED TRANSITIONAL BUTTERWORTH-CHEBYSHEV	$\gamma_{MTBC} = \gamma_C + \frac{m}{\sqrt{2}(\omega_o^2-1)} + \frac{k(k+1-2n)}{2\sqrt{2}}$ $= \gamma_{TBC} + \frac{m}{\sqrt{2}(\omega_o^2-1)}$	$\alpha_{MTBC} = \alpha_B + 6(n-m-k-1) + 20m \log\left[\frac{n-2m}{m} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right]$ $= \alpha_C - 6(m+k) + 20m \log\left[\frac{n-2m}{m} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right]$ $= \alpha_{TBC} - 6m + 20m \log\left[\frac{n-2m}{m} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right]$

From eq. (3-1) ratio of cut-off slopes of these two functions may be found as

$$r_{MTBC/B} = \frac{\gamma_{MTBC}}{\gamma_B} = n + \frac{\sqrt{2}\, m}{n(w_o^2-1)} + \frac{k(k+1-2n)}{n} \tag{3-2}$$

$r_{MTBC/B}$ v.s. w_o for n=3 through 11 and m=1,2 are given in Figures 18 and 19. Stop-band attenuation of MTBC may be written as

$$\alpha_{MTBC} = \alpha_B + 6(n-m-k-1) + 20\, m \, \mathrm{Log}\left[\frac{n-2m}{n} \cdot \frac{w_o^2-1}{w_o^2}\right] \tag{3-3}$$

Then the difference between stop-band attenuations is given by

$$d_{MTBC/B} = \alpha_{MTBC} - \alpha_B = 6(n-m-k-1) + 20\, m \log\left[\frac{n-2m}{n} \cdot \frac{w_o^2-1}{w_o^2}\right] \tag{3-4}$$

Plots of $d_{MTBC/B}$ v.s. w_o for n=3 through 11 and m=1,2 are given in FIG 20 and 21.

Figures 19-21 indicate that, MTBC filter can be made 10 times steeper than B filter still having 40 dB more attenuation at the stop-band.

C. MTBC V.S. MB FUNCTION

Using Table I, cut- off slope ratios of MTBC and MB filters may be found as

$$r_{MTBC/MB} = \frac{2m + (w_o^2-1)\left[(n-k)^2+k\right]}{2m + n(w_o^2-1)} \tag{3-6}$$

Plots of $r_{MTBC/MB}$ v.s. w_o for n=3 through 11 and m=1,2 are given in figures 22 and 23.

The difference between the stop-band attenuations of MTBC

and MB is given by

$$d_{MTBC/MB} = 6(n-k-1)$$

Equation (3-7) indicates that, stop-band attenuation difference between these filters doesn't depend on w_o. For n=10, m=1, k=1, with 48 dB more attenuation, cut-off slope ratio may be changed from 2.5 to 8.0 by changing w_o from 1.06 to 2.4.

D. MTBC FUNCTION V.S. C FUNCTION

The ratio of cut-off slope of MTBC function to cut-off slope of C function is given by

$$r_{MTBC/C} = 1 + \frac{2m}{n^2(w_o^2-1)} + \frac{k(k+1-2n)}{n^2}$$

Plot of $r_{MTBC/C}$ v.s. w_o for n=3 through 11 and m=1,2 are given in Figures 24 and 25. The difference between stop-band attenuations of MTBC and C functions is given by

$$d_{MTBC/C} = -6(m+k) + 20\, m \log\left[\frac{n-2m}{n} \cdot \frac{w_o^2-1}{w_o^2}\right]$$

Plots of $d_{MTBC/C}$ v.s w_o for n=3 through 11 and m=1,2 are given in Figures 26 and 27. The Chebyshev filter is known to provide much steeper cut-off slope than the corresponding B and TBC filters. Figure 25 shows that the cut-off slope of the MTBC filter can be made 1.4 times steeper than that of the Chebyshev filter, with n=5, m=2, and w_o=1.06.

E. MTBC FUNCTION V.S. MC FUNCTION

The ratio of cut-off slope of MTBC filter to cut-off slope of MC filter is given by

$$r_{MTBC/MC} = 1 + \frac{k(k+1-2n)(\omega_o^2-1)}{2m + n^2(\omega_o^2-1)} \qquad (3\text{-}10)$$

Plots of $r_{MTBC/MC}$ v.s. w_o for n=3 through 11 and m=1,2 are given in Figures 28 and 29.

The difference between stop-band attenuations of MTBC and MC functions is given by

$$d_{MTBC/MC} = -6k \qquad (3\text{-}11)$$

For a given k, $d_{MTBC/MC}$ is allways constant.

F. MTBC V.S. TBC FUNCTIONS

The ratio of the cut-off slopes of MTBC and TBC functions is given by

$$r_{MTBC/TBC} = 1 + \frac{2m}{(\omega_o^2-1)\left[n^2 + k(k+1-2n)\right]} \qquad (3\text{-}12)$$

Plots of $r_{MTBC/TBC}$ v.s. w_o for n=3 through 11 and m=1,2 are given in Figures 30 and 31.

The difference between the stop-band attenuaticns of these two functions is given by

$$d_{MTBC/TBC} = -6m + 20\,m \,\text{Log}\left[\frac{n-2m}{n} \cdot \frac{\omega_o^2-1}{\omega_o^2}\right] \qquad (3\text{-}13)$$

Plots of $d_{MTBC/TBC}$ v.s. w_o for n=3 through 11 and m=1,2 are given in Figures 32 and 33.

G. SUMMARY

Using the location and the order of the inserted zeros and the weighting factor as parameters, the characteristic curve of the MTBC filter can be made steeper than that of the conventional all-pole filters without greatly sacrificing either stop-band attenuation or flatness in the pass band. A Modified Chebyshev filter provides slightly better performance than the MTBC filter, at the expense of substantial degradation of the pass-band flatness. Graphs are given to help in the comparison and in determining the numerical advantages gained by increased complexity.

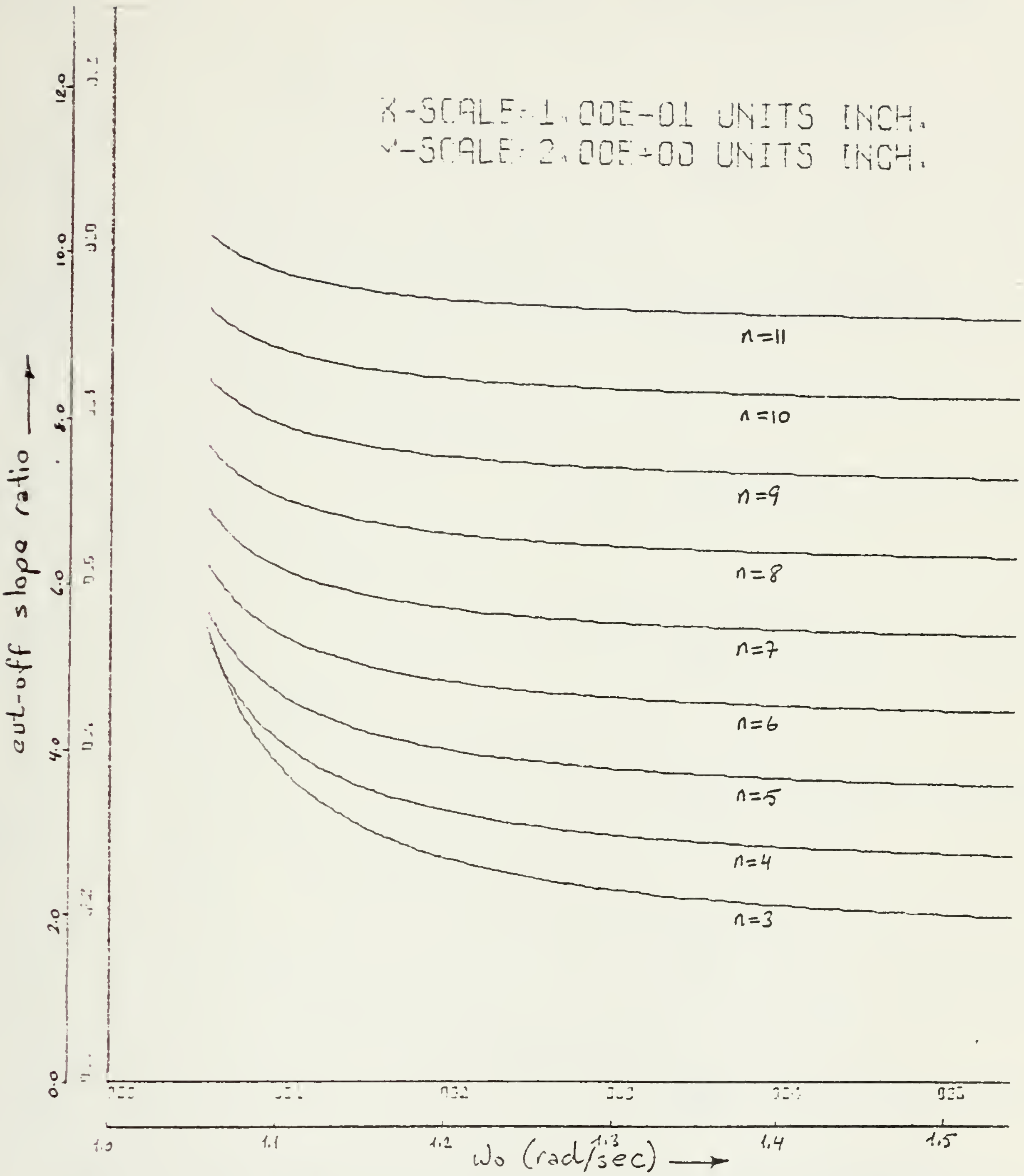

Figure 18 - RATIO OF CUT-OFF SLOPES OF MTBC AND B FUNCTIONS V.S. W (M=1)

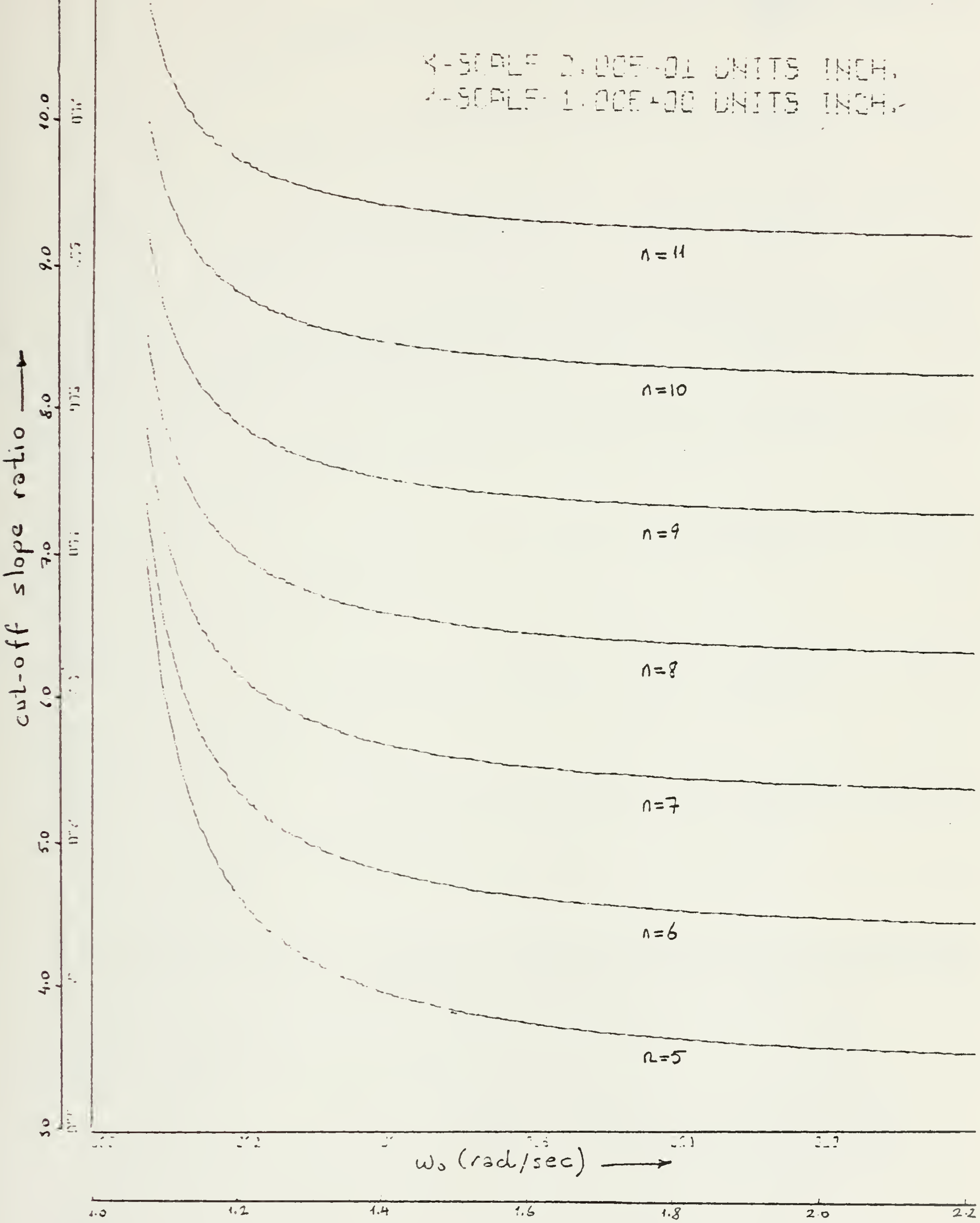

Figure 19 - RATIO OF CUT-OFF SLOPES OF MTBC AND B FUNCTIONS V.S. W (M=2)

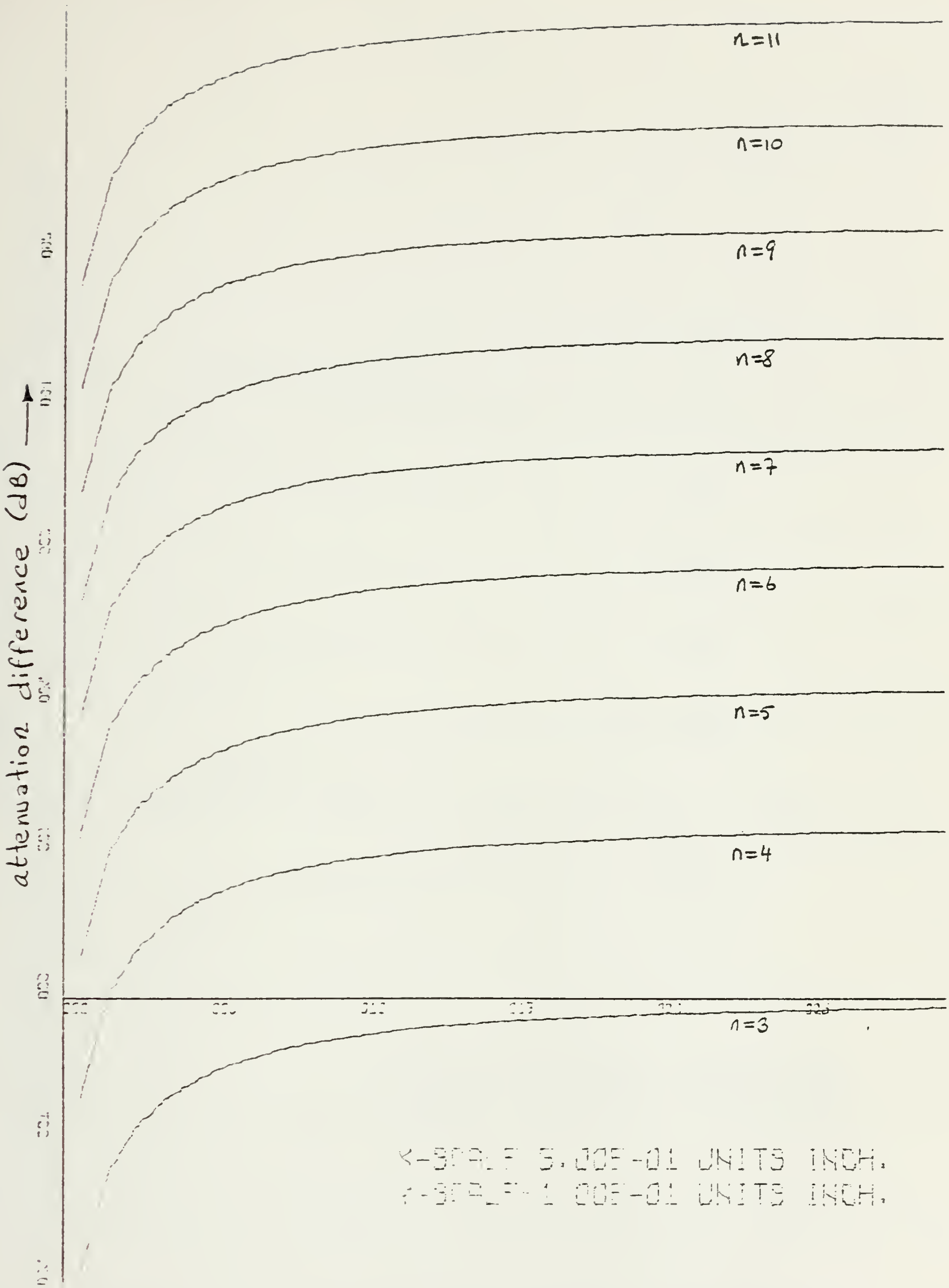

Figure 20 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND B FUNCTIONS V•S• W (M=1)

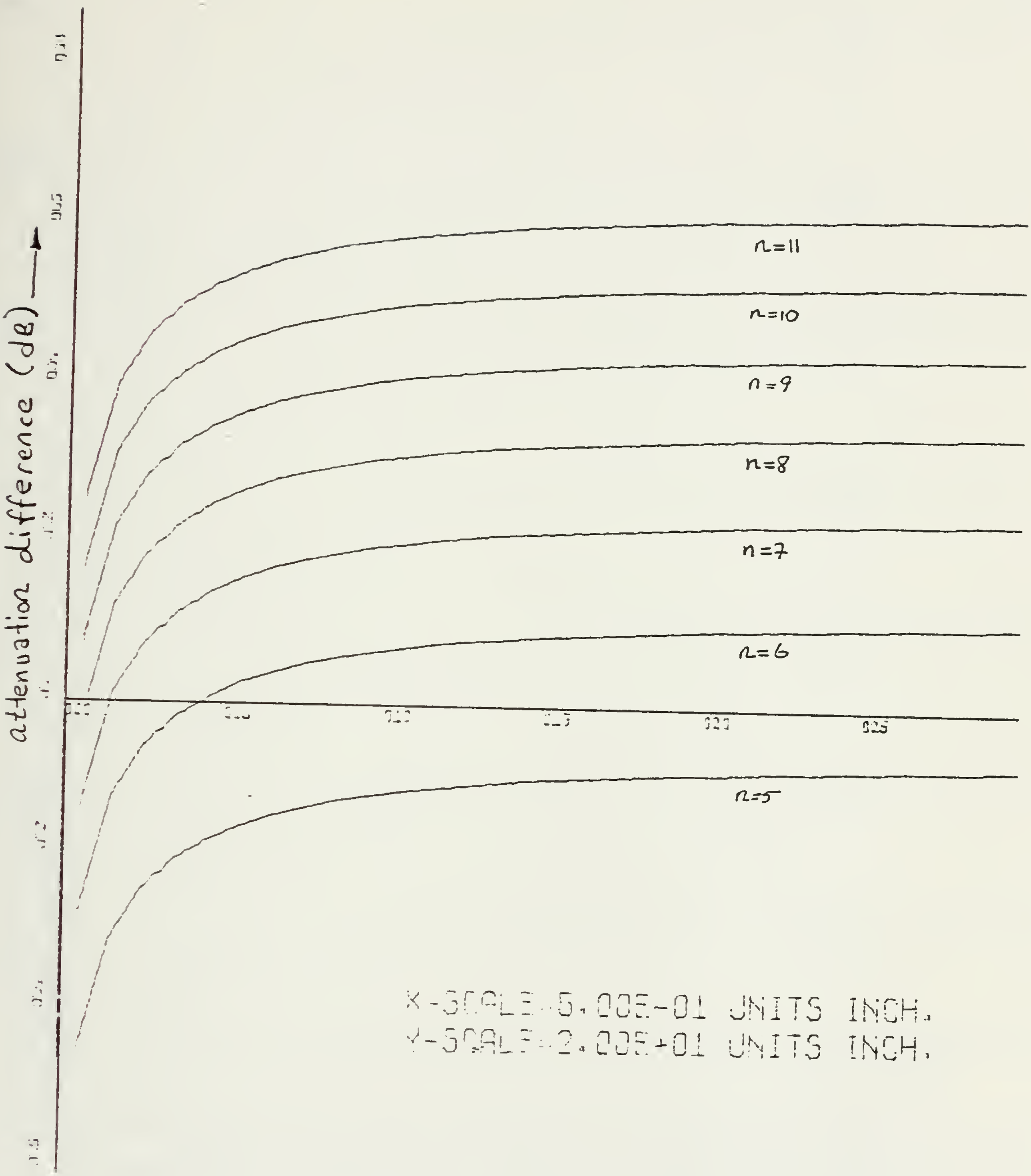

Figure 21 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND B FUNCTIONS V.S. W (M=2)

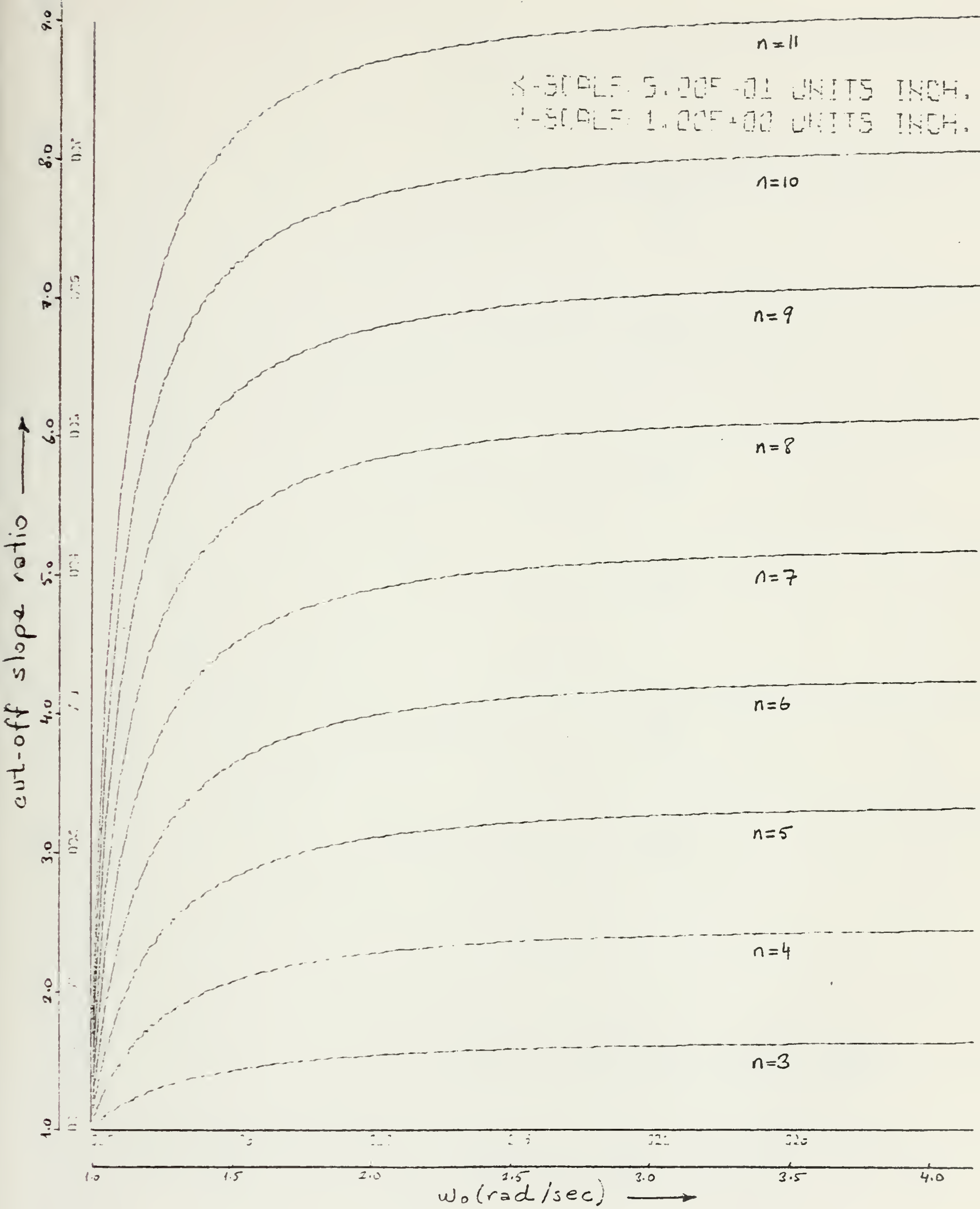

Figure 22 - RATIO OF CUT-OFF SLOPES OF MTBC AND MB FUNCTIONS V.S. W (M=1)

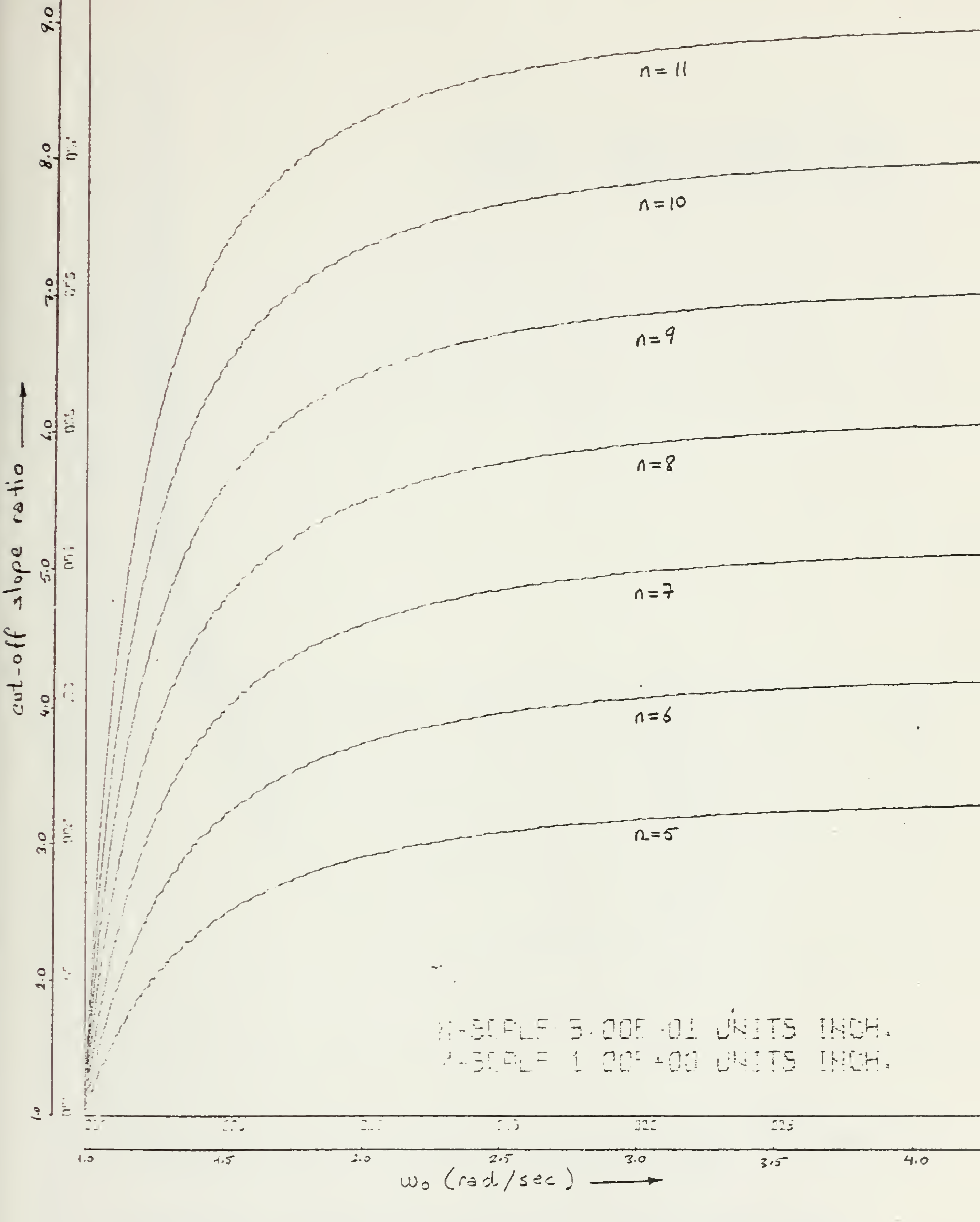

Figure 23 - RATIO OF CUT-OFF SLOPES OF MTBC AND MB FUNCTIONS V•S• W (M=2)

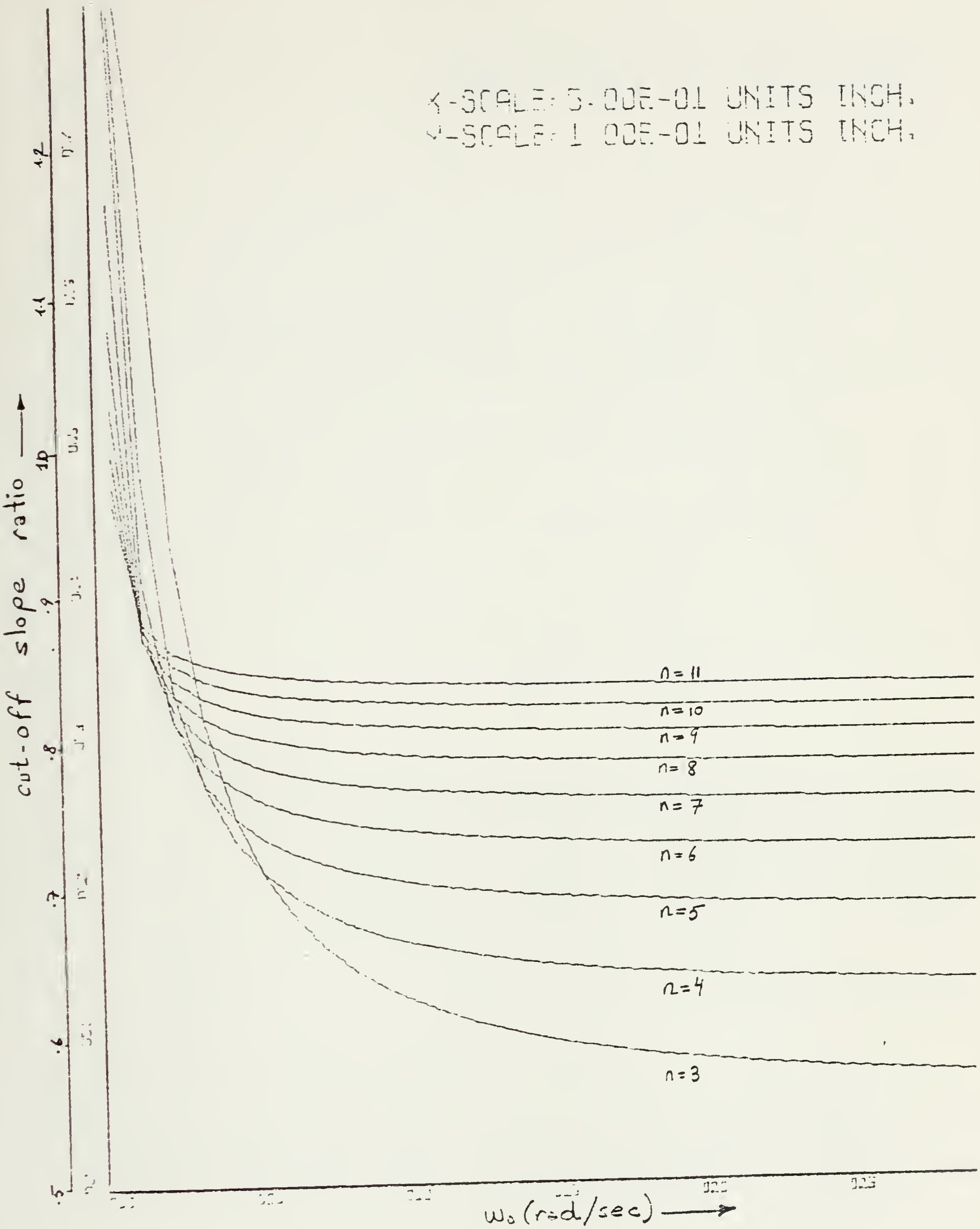

Figure 24 - RATIO OF THE CUT-OFF SLOPES OF MTBC AND C FUNCTIONS V•S• W (M=1)

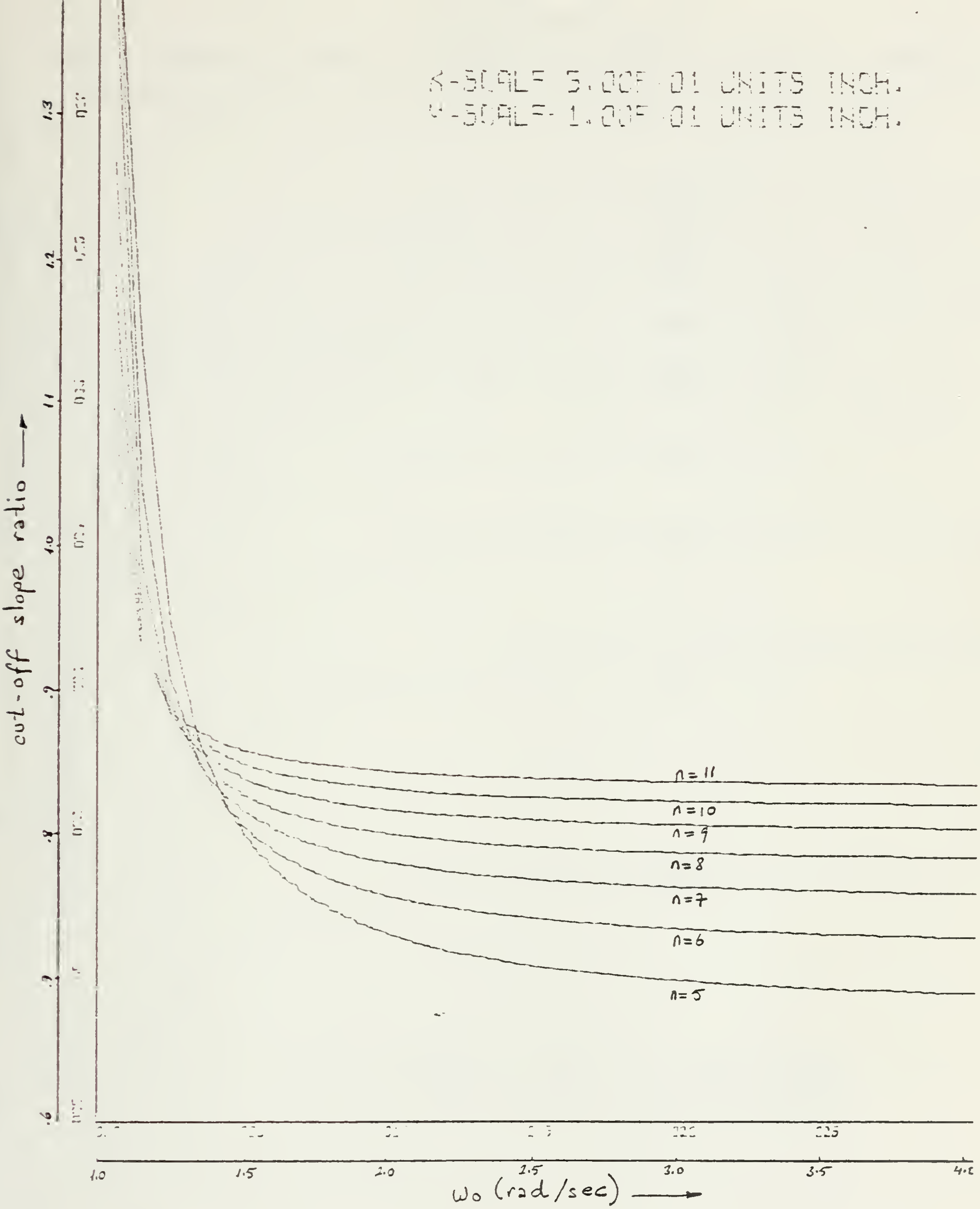

Figure 25 - RATIO OF THE CUT-OFF SLOPES OF MTBC AND C FUNCTIONS V•S• W (M=2)

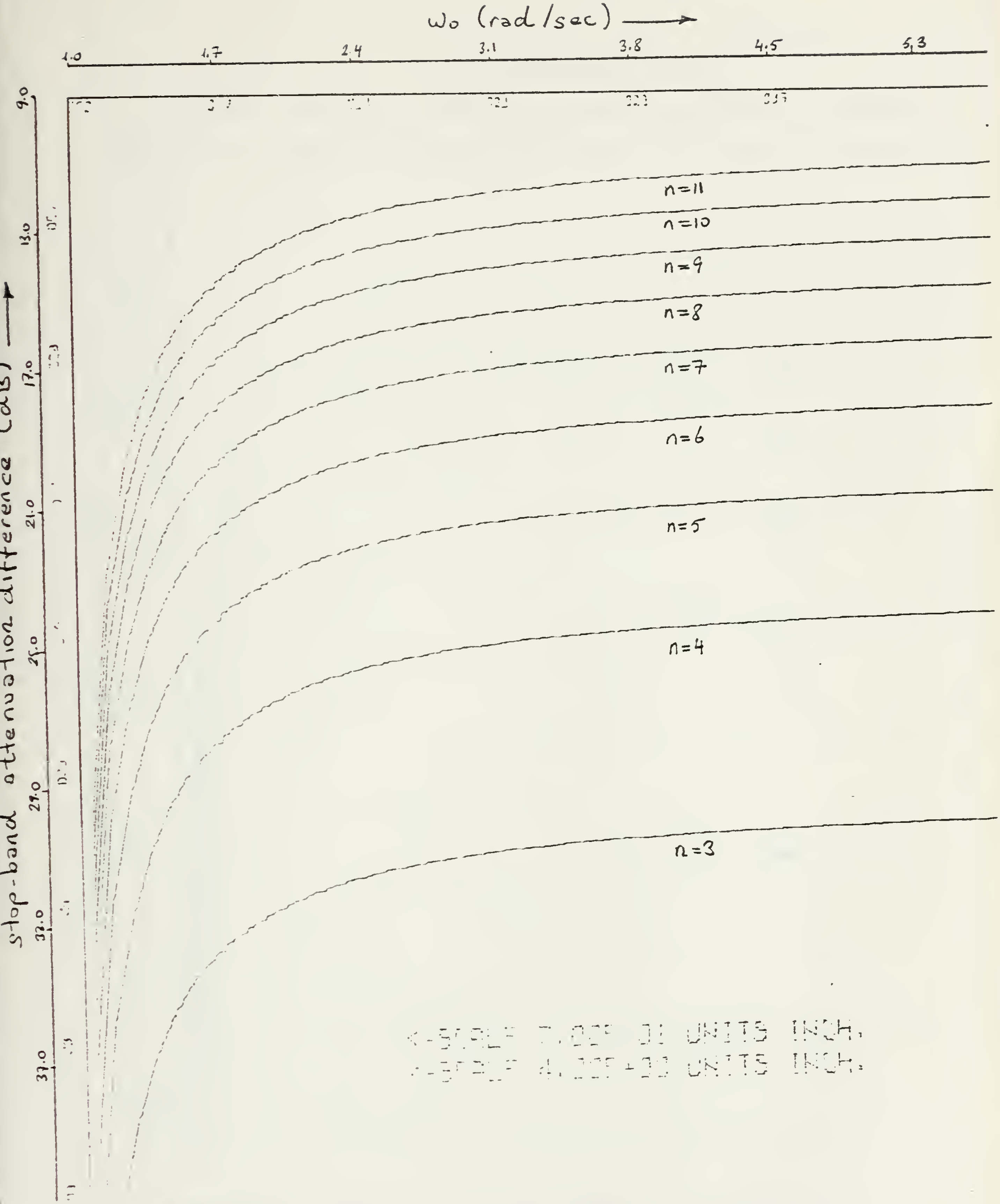

Figure 26 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND C FUNCTIONS V.S. W (M=1)

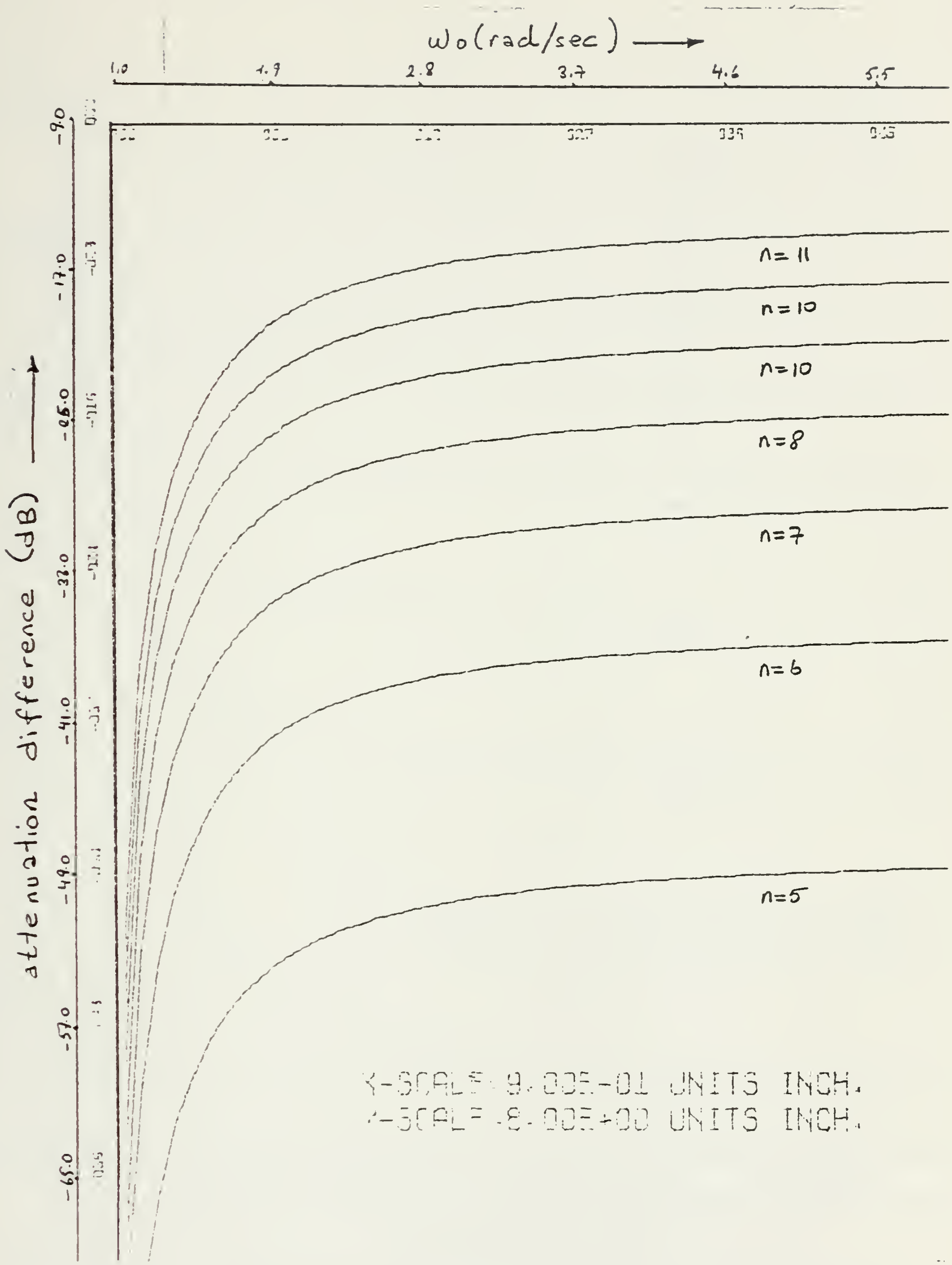

Figure 27 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND C FUNCTIONS V•S• W (M=2)

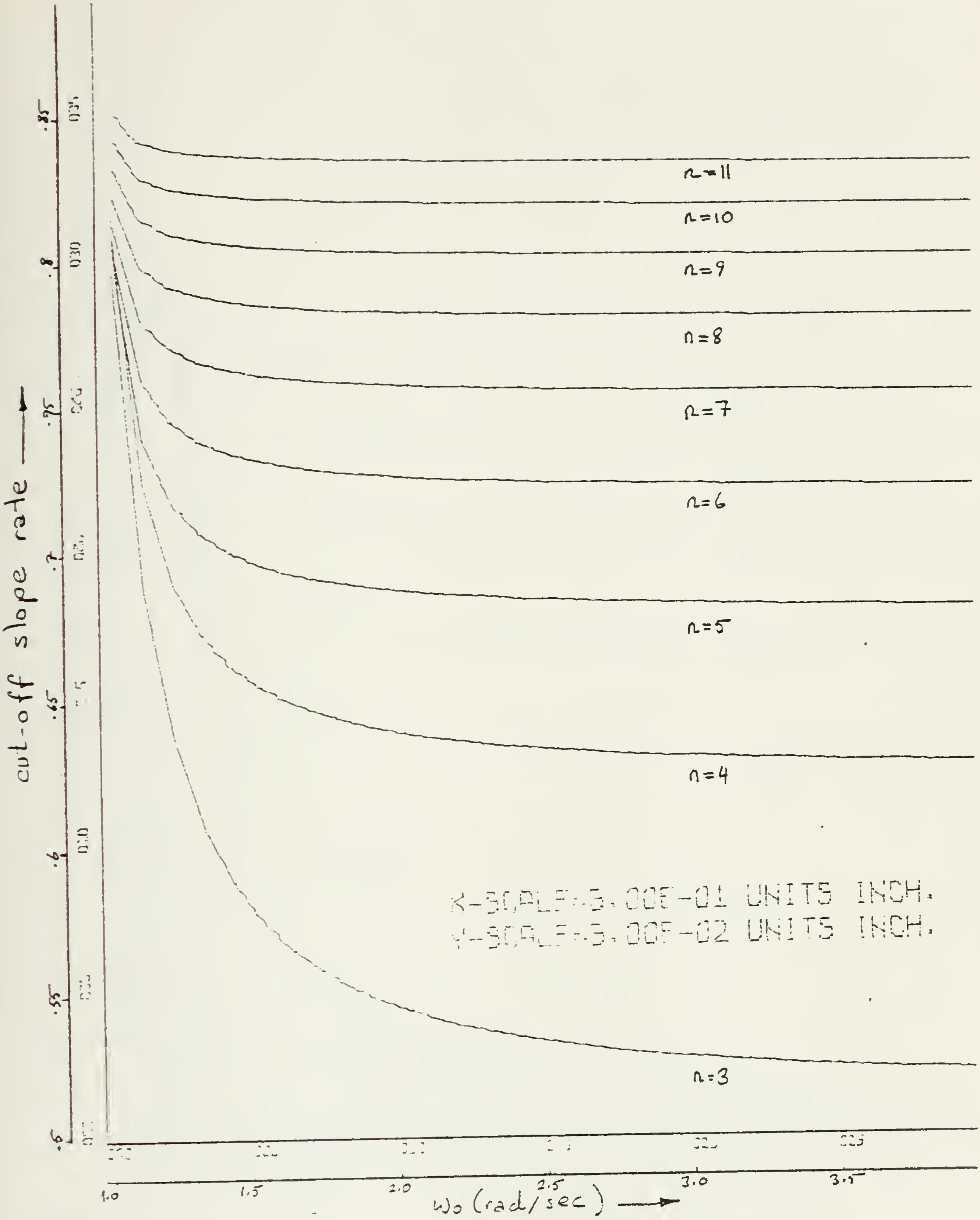

Figure 28 - RATIO OF THE CUT-OFF SLOPES OF THE MTBC AND MC FUNCTIONS V.S. W (M=1)

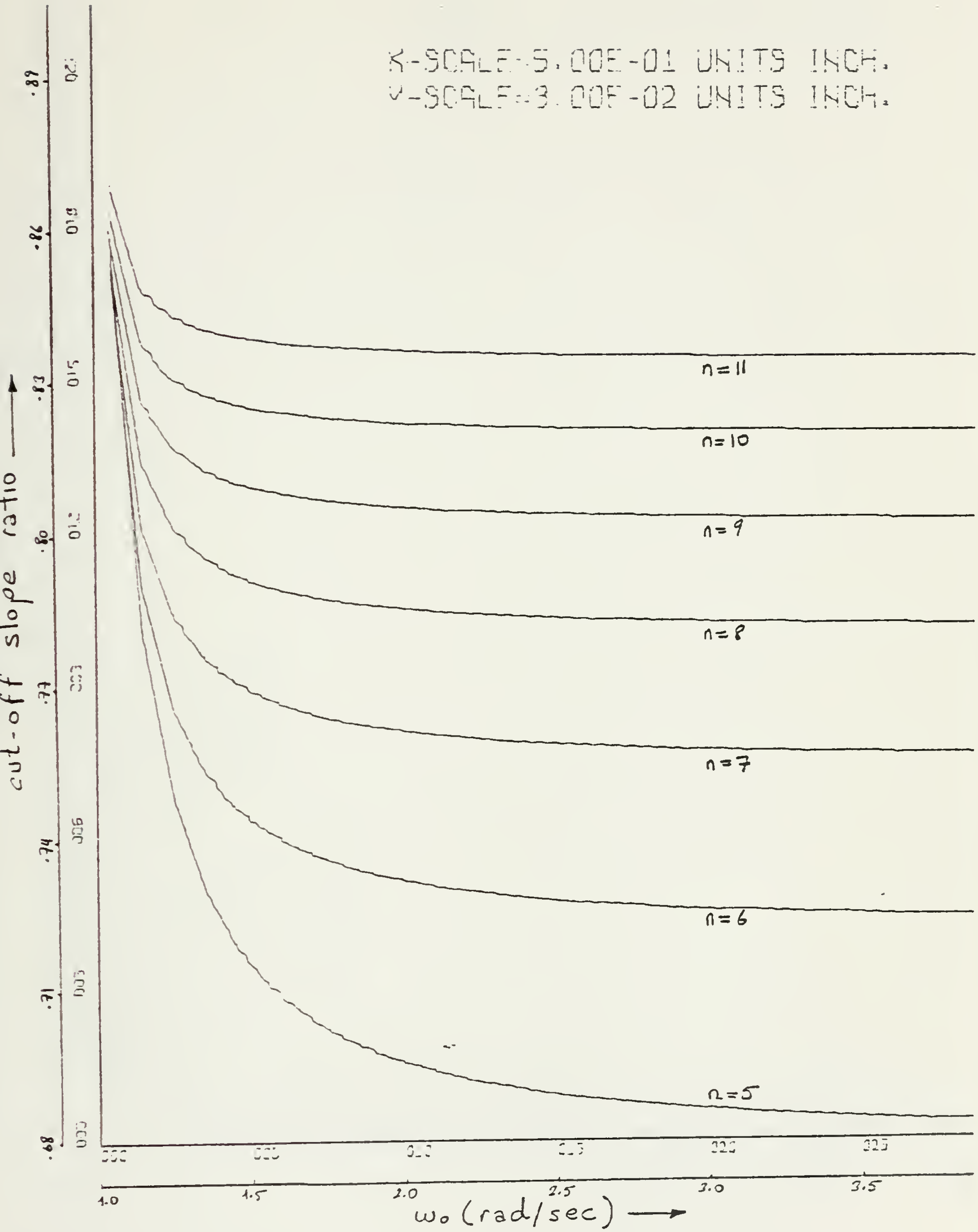

Figure 29 - RATIO OF THE CUT-OFF SLOPES OF THE MTBC AND MC FUNCTIONS V•S• W (M=2)

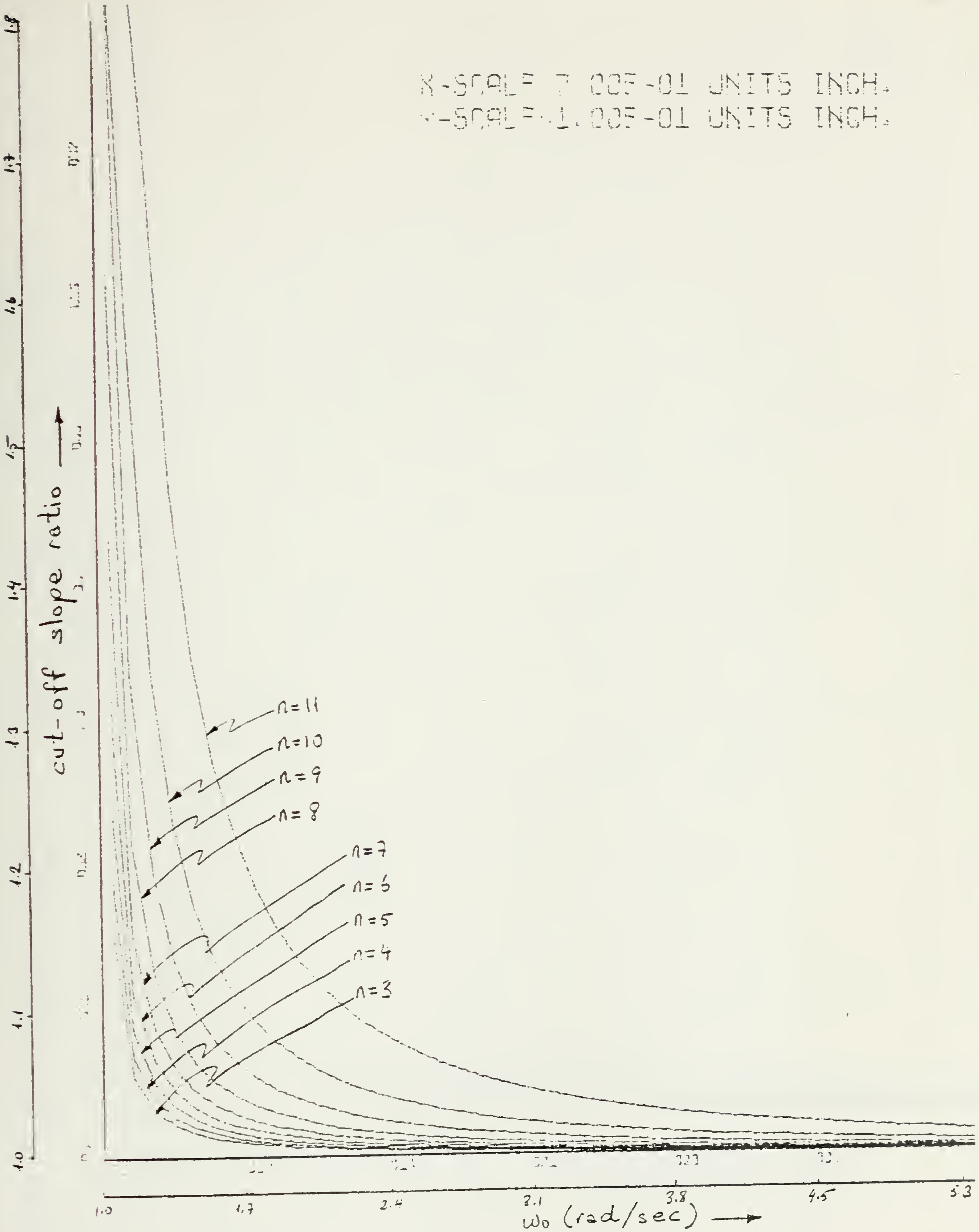

Figure 30 - RATIO OF THE CUT-OFF SLOPES OF THE MTBC AND TBC FUNCTIONS V•S• W (M=1)

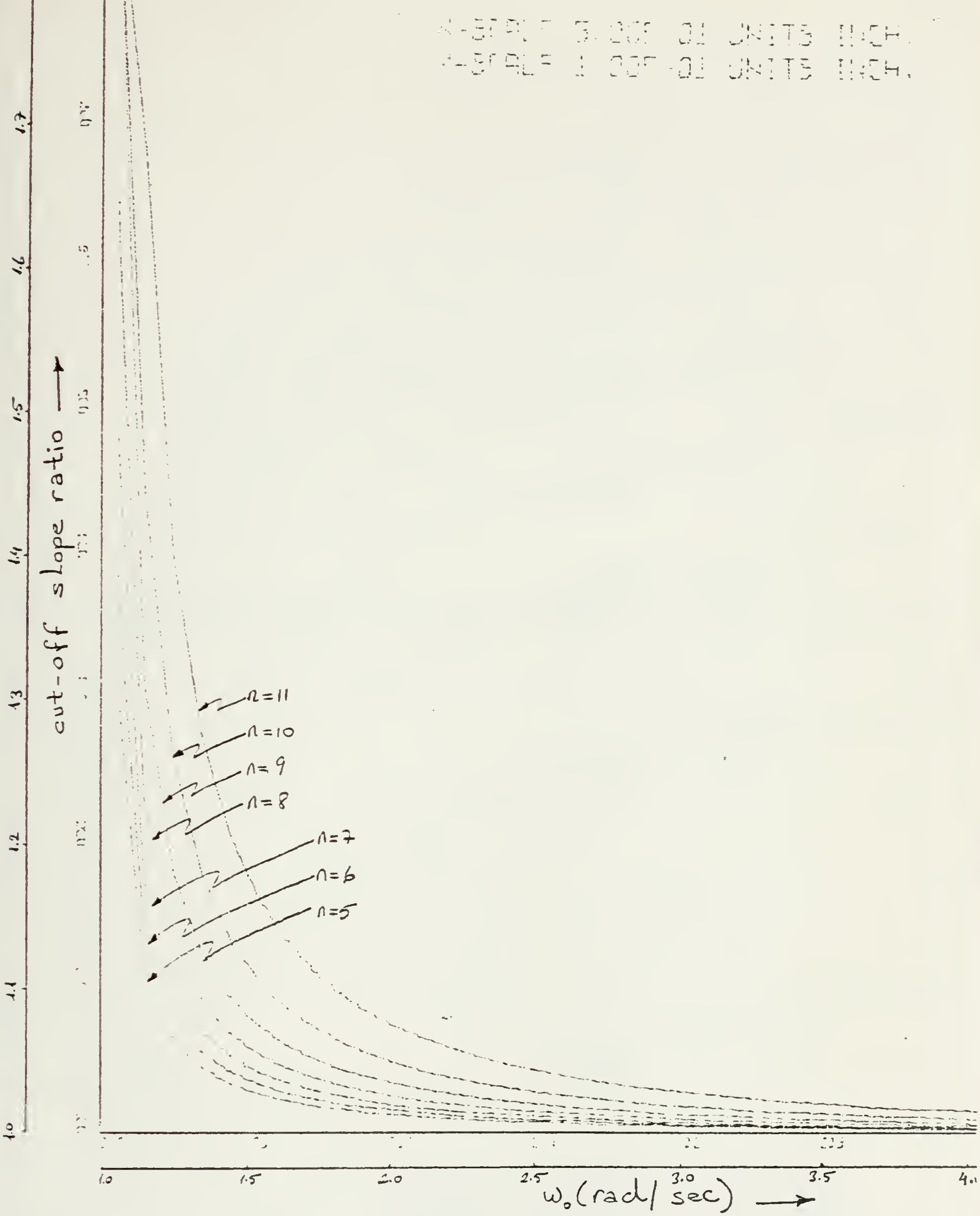

Figure 31 - RATIO OF THE CUT-OFF SLOPES OF THE MTBC AND TBC FUNCTIONS V•S• W (M=2)

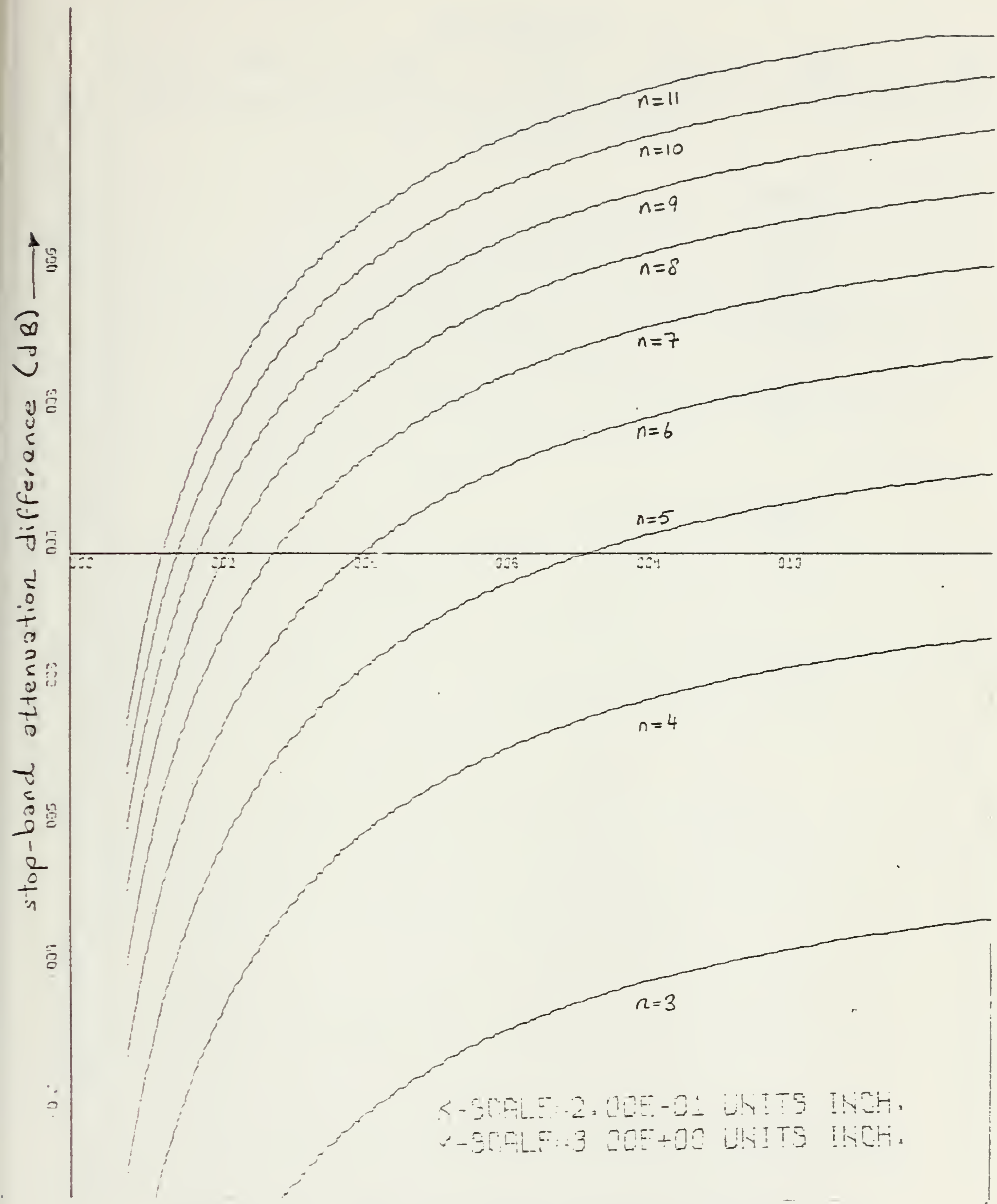

Figure 32 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND TBC FUNCTIONS V•S• W (M=1)

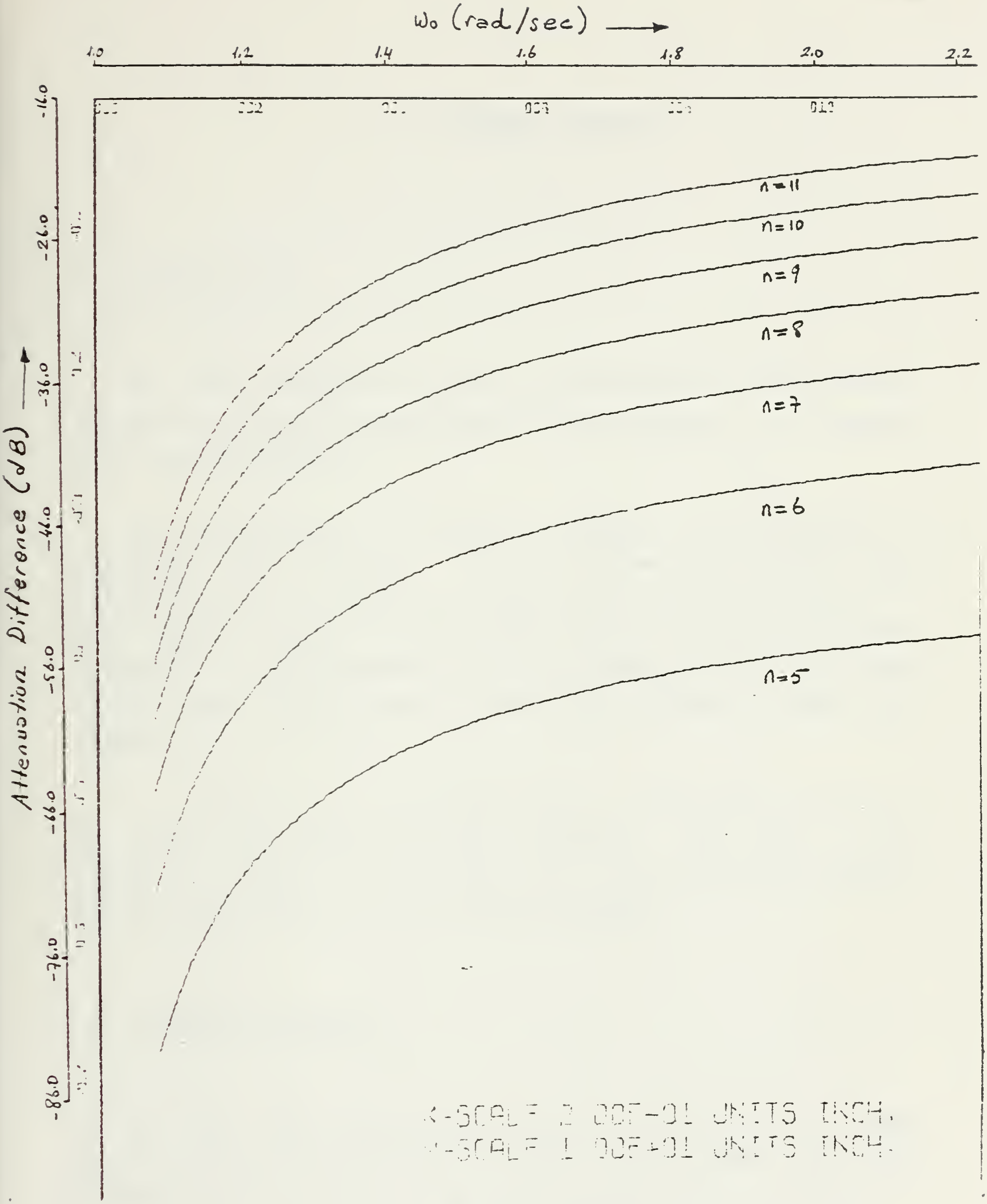

Figure 33 - THE DIFFERENCE BETWEEN THE STOP-BAND ATTENUATIONS OF MTBC AND TBC FUNCTIONS V•S• W (M=2)

IV. COMPUTER PROGRAM

A. INTRODUCTION

in the previous chapters, expressions for MTBC filter are derived and the performance of this filter is compared with various filters.

In this chapter, A computer program is developed to implement MTBC filter. It is pointed out that all five filters, which are used to compare with the MTBC filter, are the special cases of the MTBC filter. Thus the program developed in this chapter for the implementation of the MTBC filter may also be used to implement any one of these five filters.

A sample problem is worked out to illustrate the use of the plots presented in earlier chapters, the use of the computer program, and to point out the flexibility offered by the MTBC filter to the filter designer.

B. COMPUTER PROGRAM

In the most general form the transfer function of MTBC filter is given by eq.(2-27), which is repeated here for convenience.

$$|F(j\omega)|^2 = \frac{\prod_{i=1}^{m}(\omega_i^2-\omega^2)^2}{\prod_{i=1}^{m}(\omega_i^2-\omega^2)^2 + \prod_{i=1}^{m}(\omega_i^2-1)\,\omega^{2k}\,C_{n-k}^2(\omega)} \qquad (2\text{-}27)$$

where :

w = Location of i^{th} inserted zero

m = Order of inserted zeros

n = Order of filter

k = Weighting factor of Transitional Butterworth Chebyshev filters

When $k=n$, and $m=0$, $|F(jw)|^2$ is identical to the Butterworth function. With $k=n$ and $m\neq 0$, $|F(jw)|^2$ is identical to the modified Butterworth function which is discussed by BUDAK and ROY [1] - [2]. When $k=0$ and $m=0$, $|F(jw)|^2$ is identical to the Chebyshev function. With $k=0$ and $m\neq 0$ Modified Chebyshev function may be implemented which is discussed by AGARVAL and SEDRA [3].

When $m=0$ and $k\neq 0$, $|F(jw)|^2$ is identical to the Transitional Butterworth-Chebyshev filter. Finally allowing both k and m to vary, Modified Transitional Butterworth-Chebyshev filters may be implemented.

The program consists of two main parts. In the first part the analog filter, which is given by eq. (2-27), is implemented and the frequency response is plotted.

In the second part, the analog transfer function $F(jw)$ is first predistorted then transformed into z-domain by algebraic substitution method (using Bilinear z-transformation) to obtain $H(z)$. Digital transfer function [$H(z)$] is then factored into second order cascaded stages.

Finally, frequency response curves ($|F(jw)|$ v.s. w, 20 log $|F(jw)|$ v.s. w_0 and $F(jw)$ v.s. w_0) are drawn.

Bilinear Transformation is preferred over the other available algebraic substitution methods (i.e. Impulse invariant, Matched z-transformation) in obtaining H(z), mainly for the following reasons [4] :

(1) it has the property that realizable stable continuous systems are mapped to realizable stable digital filters.

(2) Wideband sharp cut-off continuous filters can be mapped to wideband sharp cut-off digital filters without the aliasing in the frequency response.

(3) After Bilinear Transformation, the relation between the analog and digital frequencies is given by

$$\omega = \frac{2}{T} \tan\left(\frac{\Omega T}{2}\right)$$

By choosing cut-off frequencies approaching to $f_s/2$ where f_s stands for sampling frequency, extremely sharp cut-off slopes may be obtained. As an example plot of cut-off slope of bilinearly transformed Butterworth filter v.s. cut-off frequency is given in FIG. 34.

C. REQUIRED DATA CARDS

The data cards required to use the program are given below.

Card 1 : Values of n,m,k in 3I2 format

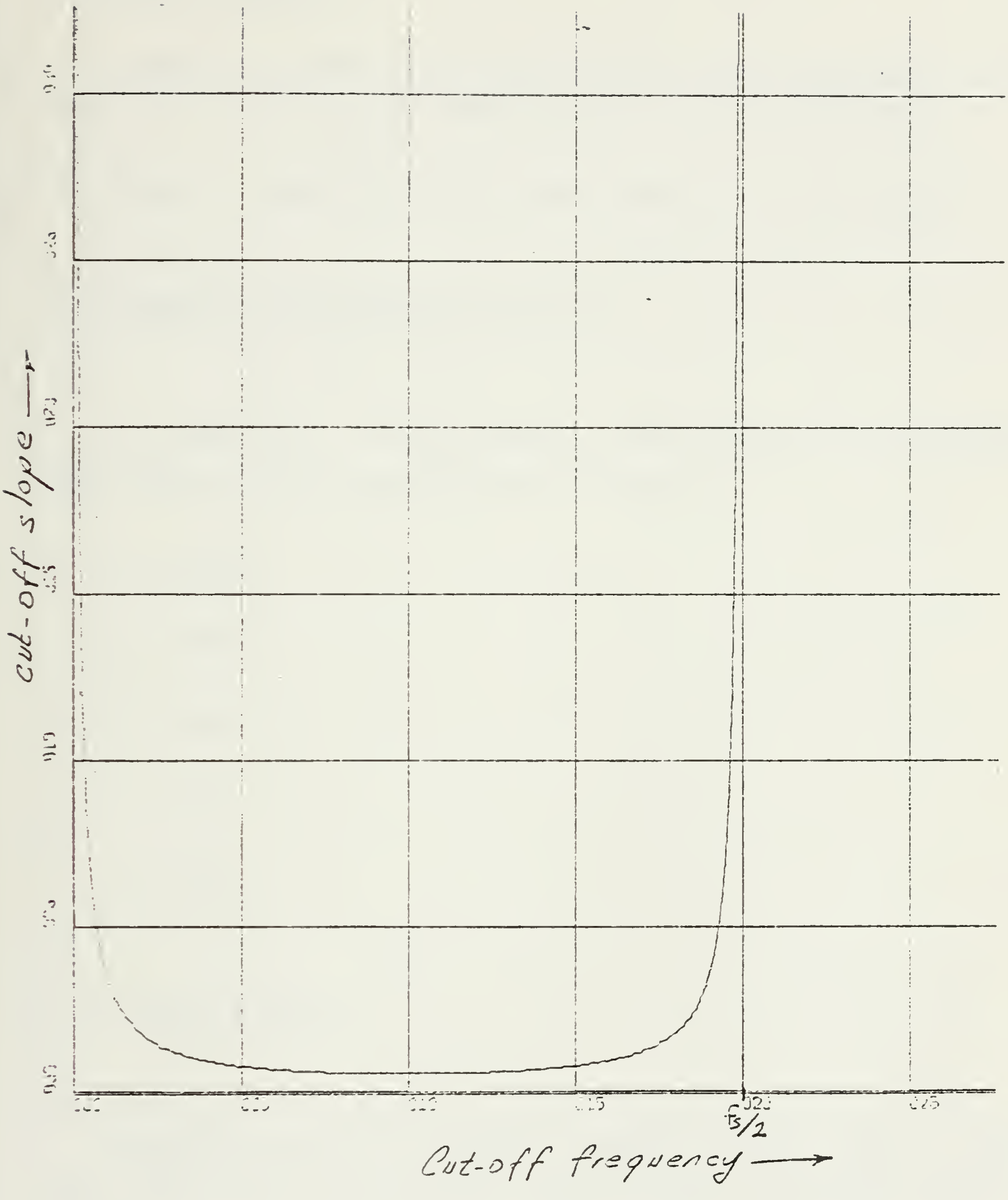

Figure 34 - CUT-OFF SLOPE OF BILINEARLY TRANSFORMED BUTTERWORTH FILTER

Card 2 : Location of inserted zeros, w_i where $i=1,2,\ldots,m$ in 8F10.5 format

Card 3 : Initial and final values of the frequency to be used for the frequency response plot in 2F10.5 format

Card 4 : Number of solutions required in I3 format

D. REQUIRED SUBROUTINES/FUNCTIONS

In addition to the built-in subroutines, the following IBM source library subroutines are used.

1. POLRT

2. PLOTP

3. PSUB

4. PMPY

5. PADDM

E. DESIGN EXAMPLE

Suppose we want to design a digital filter with a cut off slope of 15 and minumum attenuation in the stop band of 60 Db.

Possible solutions for various types of filters are given in table II.

TABLE II• POSSIBLE SOLUTIONS OF DESIGN EXAMPLE

TYPE OF FILTER	n	m	ω_0	CUT-OFF SLOPE	STOP-BAND ATTENUATION
B	43	–	–	15.2	greater than 203 dB
MB	26	2	1.16	15.0	180 dB
		3	1.26	15.05	120 dB
C	7	-	-	17.25	greater than 64 dB
MC	7	1	1.22	18.0	60 dB
		2	1.22	20.5	60 dB
TBC	8	–	–	17.75	greater than 63 dB
MTBC	7	2	1.26	15.6	60 dB
	8	1	1.16	19.8	60 dB
		2	1.19	21.1	60 dB

The following input data specifies a MTBC filter of order 7 with $m=2$, $w_0=1.36$, $k=1$, sampling period of 1 sec., 100 solution points, and frequency response plot from 0 to 3 Hz.

1	2–5	6	7–10	11–20	21–30
	7 2	1			
			1.36	1.36	
			0.0	3.0	
1	00				

Frequency responses of three of the possible solutions of the design example are given in Fig.35.

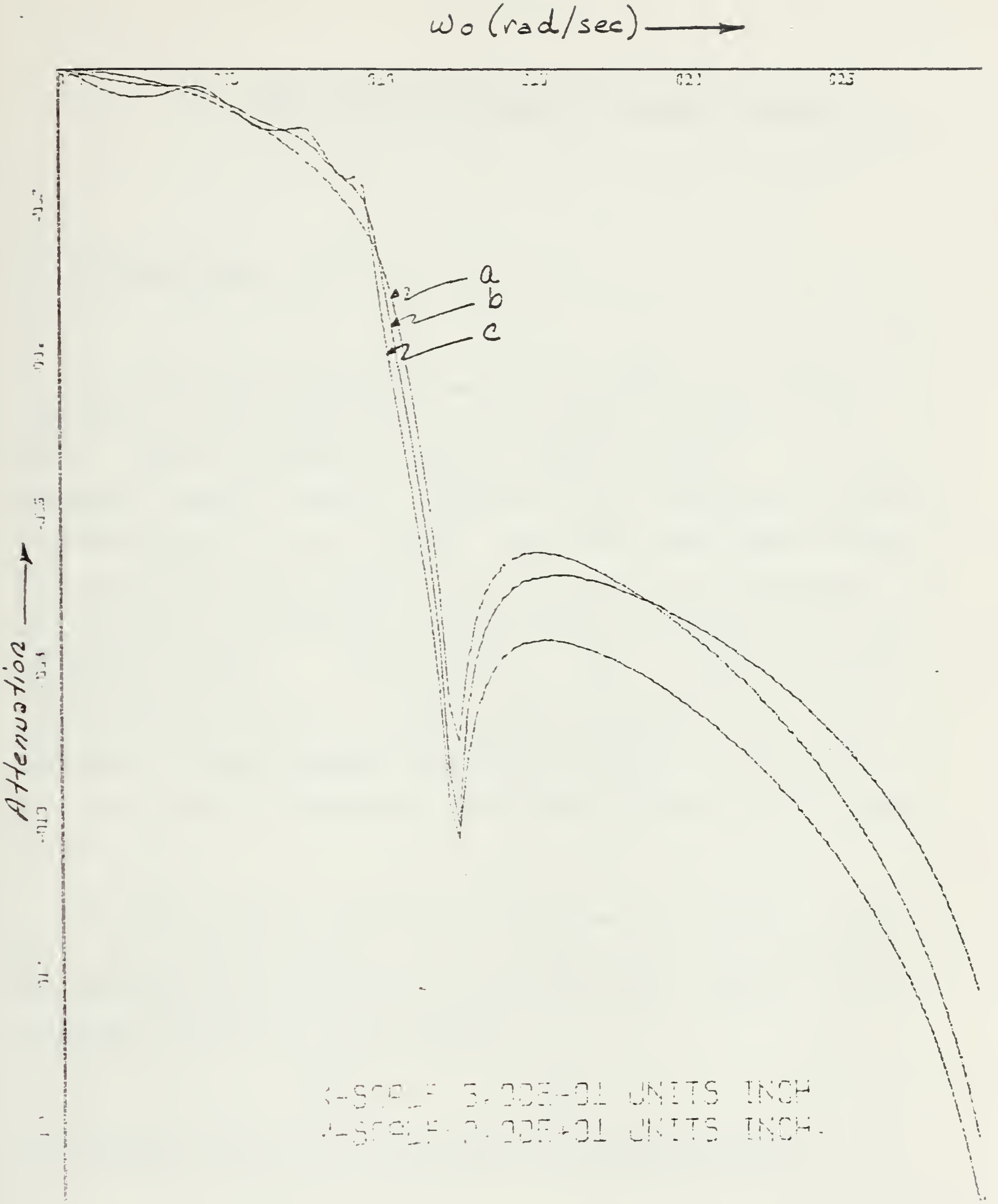

Figure 35 - MAGNITUDE RESPONSES OF (a) MB FILTER N=8, M=1, (b) MC FILTER N=7, M=1, (c) MB FILTER N=7, M=2

V. TIME DOMAIN RESPONSE OF DIGITAL FILTERS

A. INTRODUCTION

In previous chapters we have investigated the problem of desining digital filters in the frequency domain, i.e. to meet given frequency domain specifications. A filter designer should always consider the transient response characteristics of its filter. There are many applications, such as digital MTI filter, for which one is interested in the transient responses of filters that are specified in the frequency-domain. Time-domain and frequency-domain characteristics of a filter will work against each other. Filters close to the ideal frequency characteristic can be designed. Filters whose time characteristic is close to the ideal can also be designed, but filters close to both cannot [12].

In this chapter time-domain response of digital filters will be discussed and it will be shown that the location of the transfer functions poles has a profound effect on the transient response of the filter.

B. TRANSFER FUNCTIONS' POLES AND TRANSIENT RESPONSE

Given a transfer function of a digital filter in factored form

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-p_1)(z-p_2)\cdots(z-p_n)} \qquad (5\text{-}1)$$

where z_i 's and p_i 's are the zeros and poles, respectively, of the filter. In general, the input signal's Z-transform is of the form

$$X(z) = \frac{N(z)}{(z-q_1)(z-q_2)\cdots(z-q_r)} \tag{5-2}$$

where q_i 's are the poles of the input function. The response of the system may be obtained from the transfer function relationship

$$Y(z) = H(z)\cdot X(z) = K\left[\frac{(z-z_1)\cdots(z-z_m)}{(z-p_1)\cdots(z-p_n)}\right]\left[\frac{N(z)}{(z-q_1)\cdots(z-q_r)}\right] \tag{5-3}$$

For simplicity, we assume that all of the poles of the Y(z) are distinct, and making partial fraction expansion of Y(z), we obtain

$$Y(z) = k_0 + \frac{k_1 z}{z-q_1} + \cdots + \frac{k_r z}{z-q_r} + \frac{c_1 z}{z-p_1} + \cdots + \frac{c_n z}{z-p_n} \tag{5-4}$$

The response of the system may be decomposed into [5] two parts called the input signal mode (ISM) and the system mode (SM) as

$$Y_{ISM}(z) = \frac{k_1 z}{z-q_1} + \frac{k_2 z}{z-q_2} + \cdots + \frac{k_r z}{z-q_r} \tag{5-5}$$

$$Y_{SM}(z) = \frac{c_1 z}{z-p_1} + \frac{c_2 z}{z-p_2} + \cdots + \frac{c_n z}{z-p_n} \tag{5-6}$$

Combining eq. (5-4) with eq. (5-5) and eq. (5-6)

$$Y(z) = k_0 + Y_{ISM}(z) + Y_{SM}(z) \tag{5-7}$$

Eq. (5-7) indicates that the response of any linear system to any input will contain modes generated by the input

signals poles and the system transfer function poles. The time domain response of the filter may then be found by taking the inverse Z-transform of eq.(5-7)

$$y(k) = k_0 \delta k + y_{ISM}(k) + y_{SM}(k) \tag{5-8}$$

where

$$y_{ISM}(k) = k_1 (q_1)^k + k_2 (q_2)^k + \cdots + k_r (q_r)^k \tag{5-9}$$

$$y_{SM}(k) = c_1 (p_1)^k + c_2 (p_2)^k + \cdots + c_n (p_n)^k \tag{5-10}$$

Equations (5-9) and (5-10) indicate that a fast responding system is one in which all of the system transfer function poles, p_i, are sufficiently smaller than unity in magnitude, in order that the system mode will decay to zero rapidly. On the other hand, a slowly responding system is one in which the system mode decays to zero very slowly (i.e. at least one of the p is close to unity in magnitude).

Those poles closest to the unit circle will be called the system's dominant poles because they tend to dominate the characteristic of the resultant transient response.

The dominant poles' response property is, as stated by Cadzow [14] "The response time of a linear discrete system is directly dependent on the locations of the system transfer function's dominant poles. Depending on the particular response-time requirement for a given application, we then have to correspondingly locate the dominant poles of the transfer function. A fast responding system necessitates dominant poles of magnitude much less than one".

The poles of the MTBC filter for the orders 3-11 and for various values of m and w_0 are given in Table III.

TABLE III• POLES OF MTBC FILTER

n	m	Location of inserted zeros (ω_o)			
		1.06	1.08	1.1	1.12
3	1	−.172 + j.0 −.375 + j.0 −.693 + j.0	−.160 + j.0 −.516 ± j.076	−.150 + j.0 −.500 ± j.182	−.140 + j.0 −.486 ± j.240
4	1	.348 ± j.119 .057 ± j.58	.367 ± j.134 .119 ± j.601	.384 ± j.145 .165 ± j.616	.439 ± j.175 .291 ± j.651
5	1	.441 ± j.0 .355 ± j.288 .227 ± j.713	.261 ± j.0 .385 ± j.303 .269 ± j.718	.477 ± j.0 .300 ± j.721 .408 ± j.314	.489 ± j.0 .323 ± j.724 .426 ± j.322
	2	.089 + j.0 .361 + j.0 −.618 + j.0 .294 ± j.107	−.114 + j.0 −.234 + j.0 .278 + j.0 .303 ± j.136	.393 + j.0 .314 ± j.161 −.134 ± j.305	.489 + j.0 .323 ± j.724 .426 ± j.322
6	1	.479 ± j.129 .393 ± j.430 .326 ± j.765	.500 ± j.134 .421 ± j.440 .355 ± j.766	.517 ± j.137 .443 ± j.448 .376 ± j.766	.567 ± j.147 .505 ± j.470 .434 ± j.769
	2	.378 ± j.071 .236 ± j.204 −.170 ± j.563	.399 ± j.082 .260 ± j.253 −.057 ± j.625	.417 ± j.091 .285 ± j.289 .025 ± j.662	.446 ± j.106 .297 ± j.310 .087 ± j.682
7	1	.536 + j.0 .505 ± j.251 .429 ± j.529 .386 ± j.791	.555 + j.0 .527 ± j.257 .453 ± j.537 .407 ± j.790	.570 + j.0 .544 ± j.262 .471 ± j.543 .422 ± j.790	.613 + j.0 .595 ± j.274 .523 ± j.562 .464 ± j.790
	2	.426 + j.0 .377 ± j.158 .033 ± j.735 .210 ± j.360	.450 + j.0 .403 ± j.177 .115 ± j.753 .258 ± j.399	.470 + j.0 .426 ± j.192 .294 ± j.425 .174 ± j.263	.535 + j.0 .501 ± j.231 .401 ± j.489 .324 ± j.779
	3	.076 + j.0 −.374 + j.0 −.797 + j.0 .339 ± j.088 .231 ± j.136	−.103 + j.0 −.158 ± j.120 −.545 + j.0 −.301 ± j.178 −.202 + j.0	−.087 + j.0 −.143 ± j.141 −.667 ± j.187 −.305 ± j.222	−.068 + j.0 −.129 ± j.161 −.304 ± j.261 −.655 ± j.295

TABLE III• CONTINUED

n	m	Location of inserted zeros (ω_0)			
		1.06	1.08	1.10	1.12
8	1	.568 ±j.114 .456 ±j.599 .525 ±j.357 .424 ±j.805	.586 ±j.116 .475 ±j.605 .546 ±j.363 .440 ±j.804	.601 ±j.117 .490 ±j.610 .562 ±j.367 .452 ±j.804	.643 ±j.121 .610 ±j.382 .532 ±j.626 .484 ±j.804
	2	.450 ±j.081 .375 ±j.256 .166 ±j.797 .247 ±j.488	.476 ±j.088 .407 ±j.277 .227 ±j.801 .294 ±j.511	.497 ±j.093 .433 ±j.293 .328 ±j.527 .269 ±j.802	.564 ±j.107 .426 ±j.570 .513 ±j.334 .380 ±j.802
	3	.388 ±j.051 .321 ±j.146 .146 ±j.209 .348 ±j.498	.276 ±j.062 .247 ±j.187 .087 ±j.216 .325 ±j.496	.106 ±j.073 .215 ±j.196 .062 ±j.243 .127 ±j.495	−.045 ±j.091 −.364 ±j.362 −.156 ±j.251 −.649 ±j.493
9	1	.175 +j.0 .106 ±j.316 −.298 ±j.720 −.086 ±j.573 −.447 ±j.816	.201 +j.0 .131 ±j.329 −.290 ±j.736 −.438 ±j.825 −.069 ±j.593	.223 +j.0 .153 ∓j.337 −.285 ±j.752 −.432 ±j.833 −.057 ±j.607	.238 +j.0 .166 ±j.347 −.279 ±j.760 −.428 ±j.838 −.047 ±j.622
	2	.293 ±j.572 .490 ±j.0 .463 ±j.165 .383 ±j.352 .254 ±j.823	.516 ±j.0 .491 ±j.176 .418 ±j.370 .333 ±j.586 .300 ±j.821	.076 +j.0 −.152 ±j.451 .014 ±j.246 −.351 ±j.601 −.519 ±j.732	.623 +j.0 .493 ±j.637 −.326 ±j.480 −.216 ±j.435 −.725 ±j.247
	3	−.075 +j.0 −.118 ±j.127 −.461 ±j.302 −.242 ±j.224 −.719 ±j.444	−.056 +j.0 −.082 ±j.243 −.437 ±j.368 −.226 ±j.251 −.682 ±j.478	−.017 +j.0 −.206 ±j.310 −.067 ±j.171 −.413 ±j.423 −.641 ±j.563	−.007 +j.0 −.190 ±j.341 −.045 ±j.187 −.396 ±j.462 −.613 ±j.601

TABLE III. CONTINUED

n	m	Location of inserted zeros (ω_0)			
		1.06	1.08	1.1	1.12
10	1	.468 ± j.820 .488 ± j.686 .630 ± j.100 .605 ± j.305 .548 ± j.510	.228 ± j.161 −.310 ± j.773 .102 ± j.464 −.114 ± j.670 −.434 ± j.842	.248 ± j.165 −.105 ± j.686 −.305 ± j.784 .119 ± j.476 −.429 ± j.848	.264 ± j.168 −.098 ± j.698 −.302 ± j.793 .132 ± j.487 −.425 ± j.852
	2	.514 ± j.080 −.472 ± j.248 −.333 ± j.629 −.399 ± j.432 −.314 ± j.835	.362 ± j.096 −.426 ± j.462 −.226 ± j.596 −.182 ± j.407 −.416 ± j.792	.103 ± j.125 −.360 ± j.659 −.181 ± j.536 −.004 ± j.358 −.501 ± j.768	.076 ± j.142 −.276 ± j.712 −.143 ± j.502 .047 ± j.301 −.562 ± j.733
	3	−.057 ± j.071 −.283 ± j.306 −.134 ± j.201 −.480 ± j.416 −.667 ± j.550	−.052 ± j.102 −.273 ± j.308 −.143 ± j.212 −.512 ± j.420 −.661 ± j.543	−.049 ± j.112 −.269 ± j.307 −.151 ± j.220 −.516 ± j.426 −.658 ± j.533	−.043 ± j.119 −.263 ± j.312 −.160 ± j.226 −.521 ± j.430 −.655 ± j.530
11	1	.255 + j.0 .201 ± j.299 −.331 ± j.787 −.436 ± j.848 .040 ± j.554 −.165 ± j.708	.280 + j.0 .226 ± j.309 −.326 ± j.799 −.430 ± j.854 .058 ± j.572 −.156 ± j.726	.310 + j.0 .235 ± j.317 −.319 ± j.802 −.425 ± j.861 .067 ± j.593 −.143 ± j.745	.331 + j.0 .241 ± j.321 −.313 ± j.811 −.419 ± j.868 .072 ± j.602 −.138 ± j.752
	2	.093 + j.0 .062 ± j.226 −.243 ± j.571 −.512 ± j.813 −.391 ± j.651 −.061 ± j.412	.126 ± j.0 .078 ± j.232 −.224 ± j.579 −.500 ± j.780 −.380 ± j.683 −.053 ± j.433	.154 + j.0 .105 ± j.245 −.209 ± j.602 −.368 ± j.702 −.486 ± j.794 −.032 ± j.455	.171 + j.0 .116 ± j.251 −.197 ± j.615 −.471 ± j.791 −.357 ± j.713 −.024 ± j.472
	3	−.018 + j.0 −.157 ± j.277 −.640 ± j.622 −.053 ± j.146 −.480 ± j.500 −.312 ± j.388	.032 + j.0 −.131 ± j.311 −.610 ± j.651 −.031 ± j.162 −.451 ± j.603 −.286 ± j.415	.051 + j.0 −.105 ± j.344 .010 ± j.184 −.577 ± j.694 −.264 ± j.470 −.429 ± j.579	.076 + j.0 −.087 ± j.361 −.543 ± j.709 .004 ± j.205 −.242 ± j.505 −.416 ± j.641

Although this partial fraction expansion method helps us to understand the importance of the poles of the systems transfer function in transient response analysis of digital filters, evaluation of residues of corresponding poles of partial fraction expansion is not a trivial problem.

We believe that the so called ' transfer matrix ' method, which we are about to discuss, is more suitable for digital computer simulation.

Given a digital filter weighting sequence h(n) and the input sequence x(n), the response of the digital filter may be obtained by convolution summation

$$y(n) = \sum_{k=0}^{n} h(n-k)\, x(k) = \sum_{k=0}^{n} h(k)\, x(n-k) \qquad (5\text{-}11)$$

Eq. (5-11) may be written in matrix form as

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(n) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & 0 & \cdots & 0 \\ h(1) & h(0) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ h(n) & h(n-1) & h(n-2) & \cdots & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(n) \end{bmatrix} \qquad (5\text{-}12)$$

Or

$$\underline{y} = \underline{G} \cdot \underline{x} \qquad (5\text{-}13)$$

where G is the systems transfer matrix, which is defined as

$$\underline{G} = \begin{bmatrix} h(0) & 0 & 0 & \cdots & 0 \\ h(1) & h(0) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ h(n) & h(n-1) & h(n-2) & \cdots & h(0) \end{bmatrix} \qquad (5\text{-}14)$$

To find the systems response using eq. (5-13), systems transfer matrix must be available. To obtain the weigting

sequence (impulse response) of the system to form the system transfer matrix in terms of the transfer function's coefficients, the difference equation of the system is solved for the impulse input, i.e.,

$$h(n) = a_0 - \sum_{i=1}^{m} b_i \, y(n-i) \qquad , \quad n \geqslant 0 \qquad (5\text{-}15)$$

where a_i and b_i are the numerator and denominator coefficients, respectively, of the transfer function of the digital filter.

A computer program(FORTRAN) is developed to investigate the transient response of the digital filters, using equations (5-13) and (5-15). Program listing is given in Appendix B.

Step responses of MTBC filter for various values of n, m, and w_0 are given in figures 36-39. Figure 36 and 37 indicate that increasing the values of m and w_0 also increases the overshoot and settling time, but doesnt have any significant effect on the rise time. The rise time tends to increase with increasing order of the filter.

C. SUMMARY

Transient responses of digital filters depend on the position of the poles of its transfer function. A fast responding filter has poles of magnitude much less than one. Modification of all pole filters increase settling time, decrease peak overshoot, and doesnt significantly affect the rise time of the filter's step response.

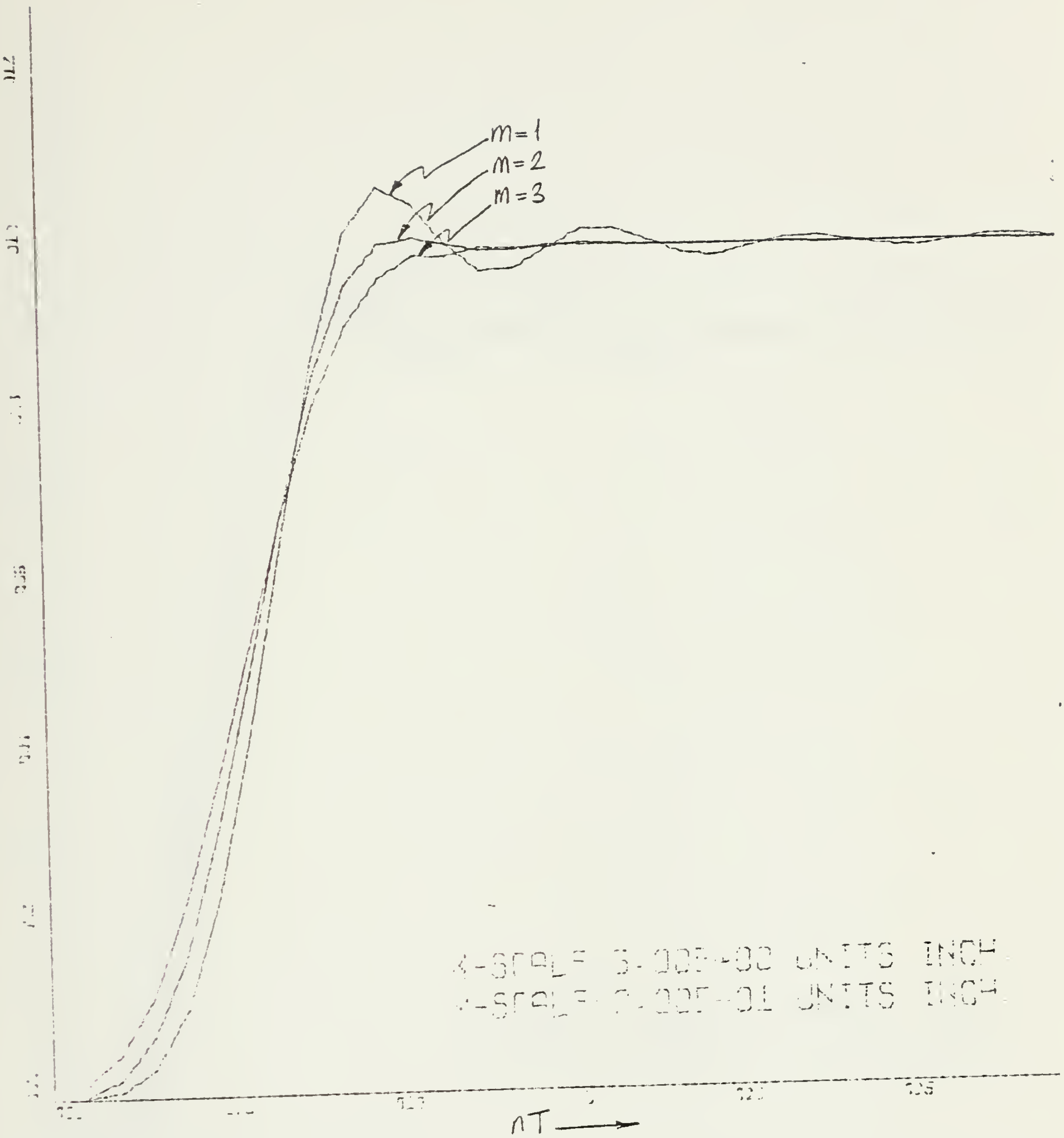

Figure 36 - STEP RESPONSE OF MTBC FILTER(N=7, W =1•46)

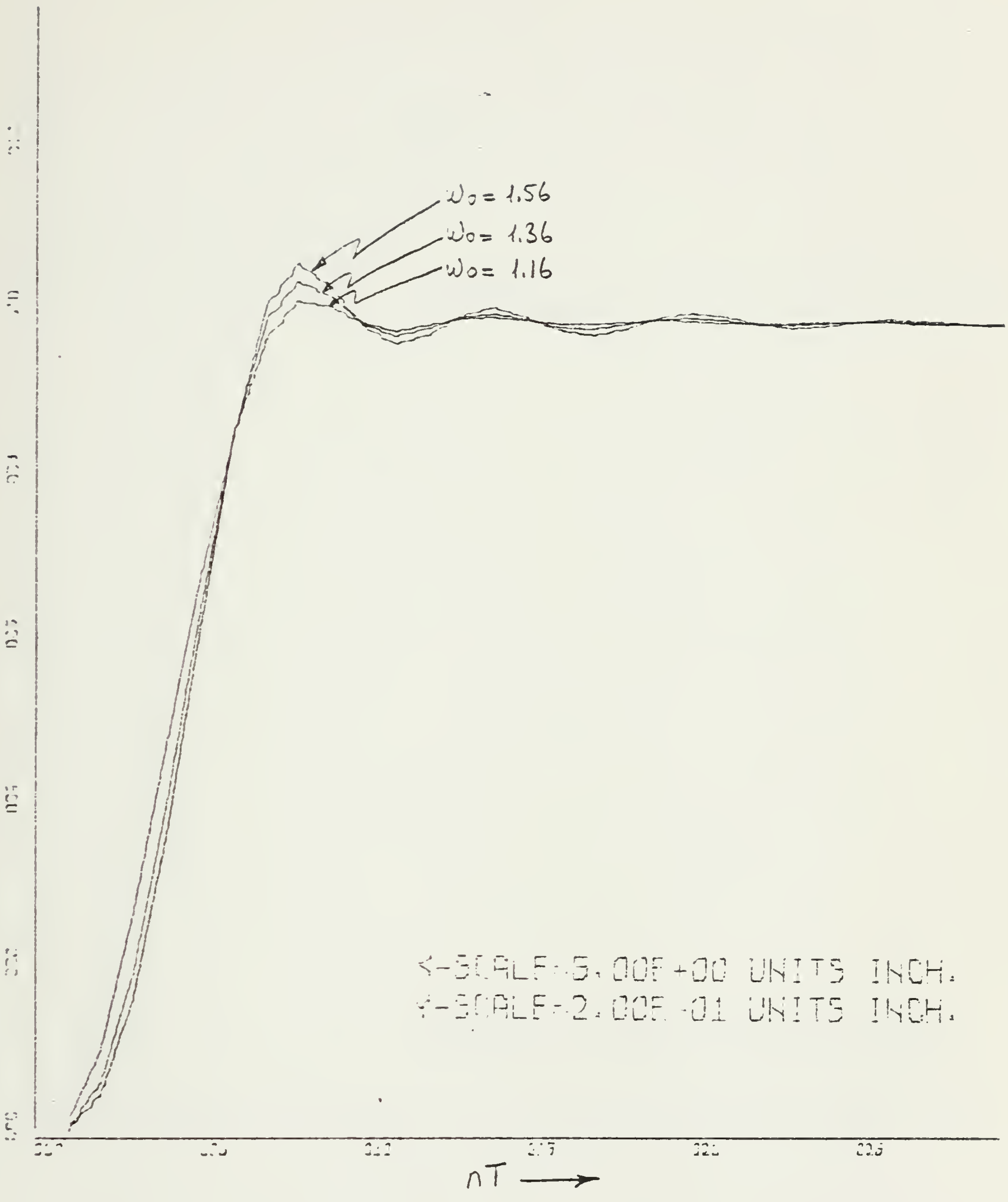

Figure 37 - STEP RESPONSE OF TMBC FILTER(N=5, M=1, K=1)

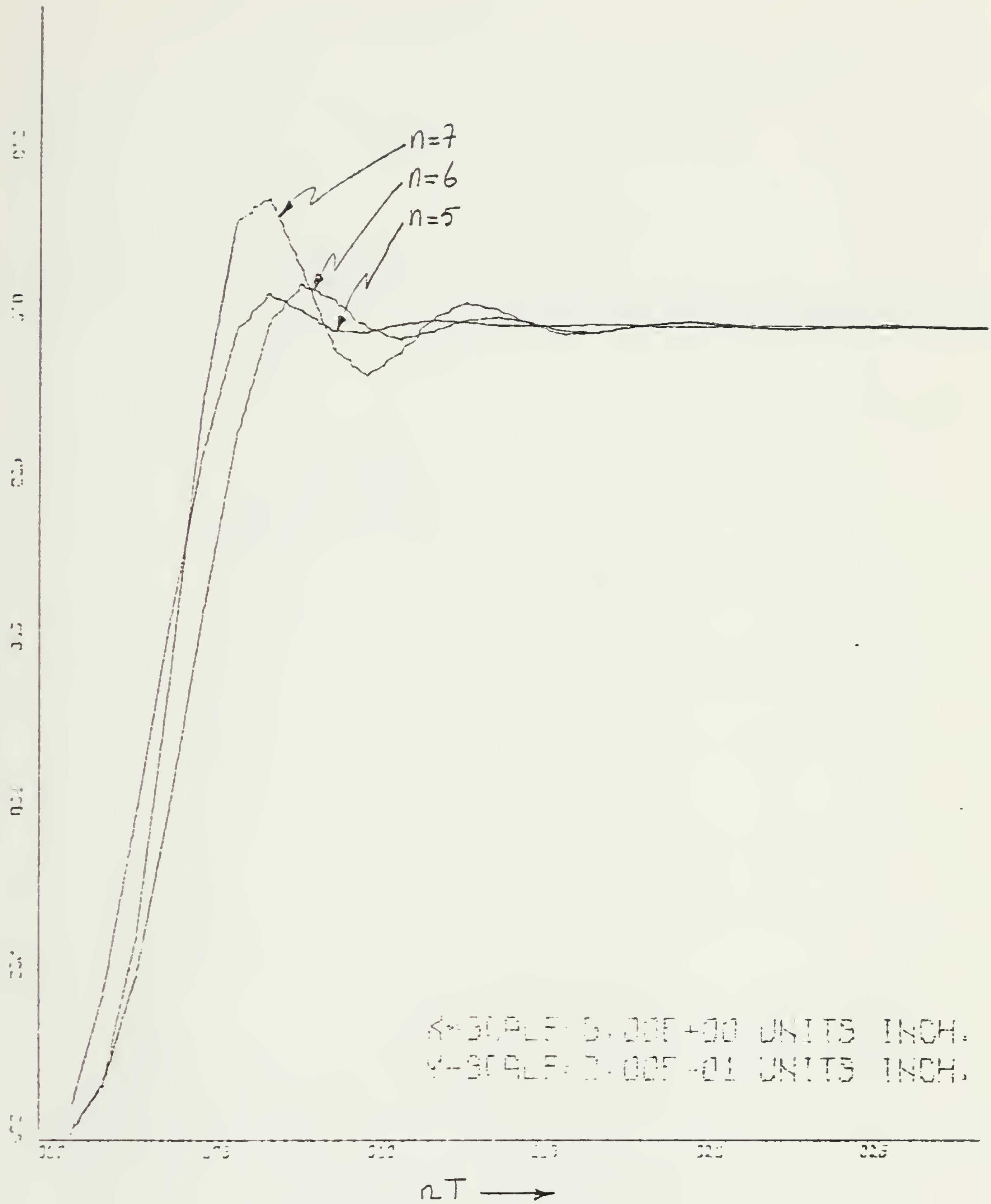

Figure 38 - STEP RESPONSE OF MTBC FILTER(M=1, W =1•36)

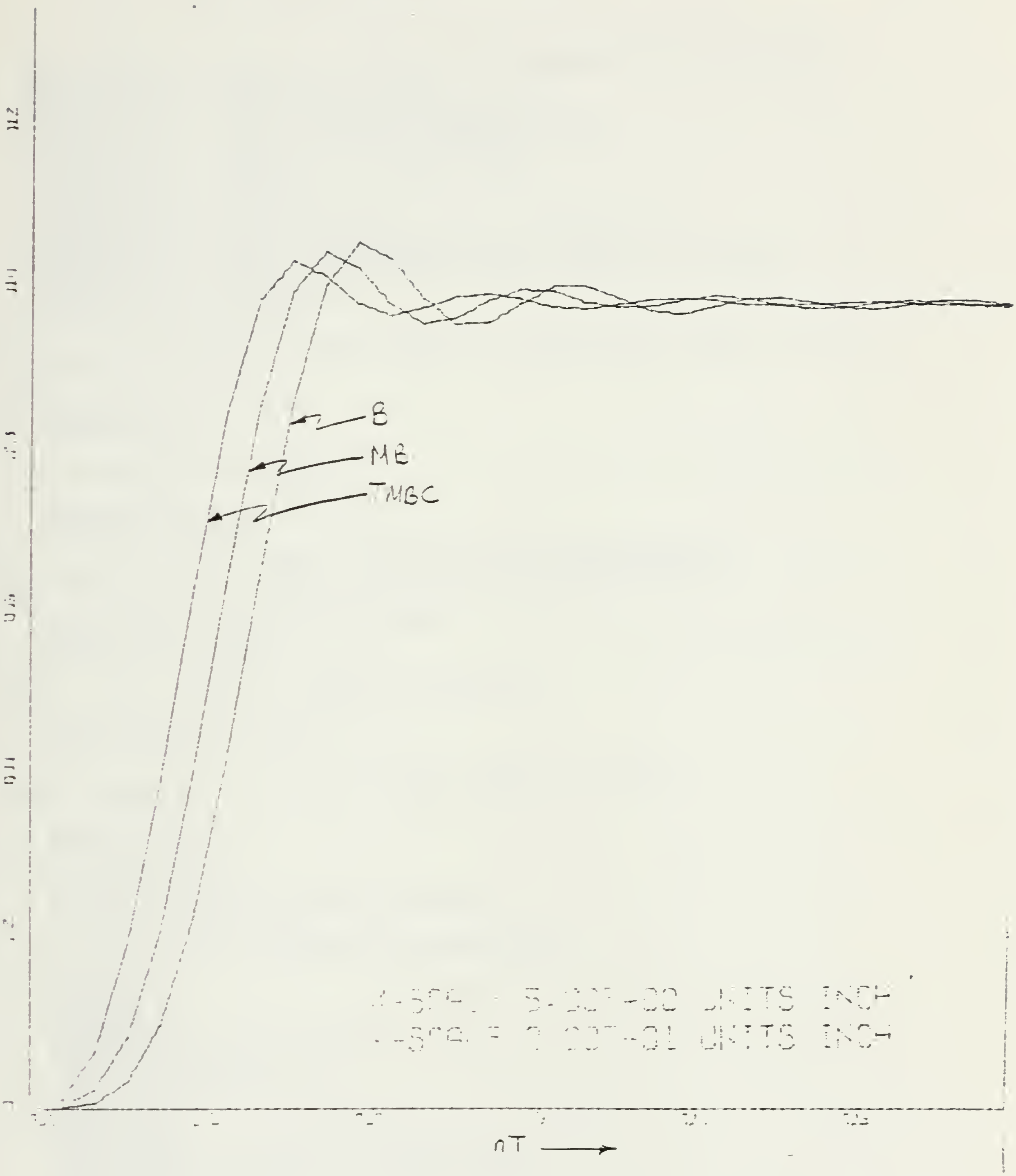

Figure 39 - STEP RESPONSES OF B, MB, MTBC FILTERS(N=5, M=1, W =1•36)

APPENDIX A

COMPUTER PROGRAM LISTING

```
      DIMENSION RNUMR(25),RNUMI(25),RDNUMR(25),RDNUMI(25),
     *C1(25),C(25),R1(25),R(25)
      DIMENSION X(50),Z(50),W(50),Y(50)
      DIMENSION ROOTR(50),ROOTI(50),COF(50)
      DIMENSION CM(50),CY(50),CZ(50)
      DIMENSION  ROOTDR(50),ROOTDI(50)
      DIMENSION CY1(50)
      DIMENSION PX(100),PY(100),RC(100),RC1(100)
      DIMENSION RN(25),RD(25)
      DIMENSION XT(25),YT(25),AX(25),AY(25)
      DIMENSION DUMMY(100),DUMMX(100),ITB(20),RTB(30)
      DIMENSION DUMKAY(100)
      COMPLEX R1,R,C,C1
C
C  READ THE ORDER OF FILTER,ORDER OF INSERTED ZEROS AND THE
C  WEIGHTING FACTOR
C
      READ(5,501) N,M,K
  501 FORMAT(3I2)
C
C  READ LOCATION OF ZEROS
C
      READ(5,500) (W(I),I=1,M)
  500 FORMAT(8F10.5)
C
C READ INITIAL AND FINAL VALUES OF THE FREQUENCY
C  (TO BE USED IN FREQUENCY RESPONSE CALCULATIONS)
C
      READ(5,502) WBEGIN,WLAST
  502 FORMAT(2F10.5)
C
C  READ THE NUMBER OF SOLUTION POINTS
C
      READ(5,509) NPOINT
  509 FORMAT(I3)
C
C  READ SAMPLING PERIOD OF THE DIGITAL FILTER
C
      READ(5,70) T
   70 FORMAT(F10.5)
C
      XPOINT=NPOINT
      WDELTA=(WLAST-WBEGIN)/XPOINT
C
C  CALCULATION OF NUMERATOR POLYNOMIAL
C
      IDIMY=1
      Y(1)=1.0
      IF(M.EQ.0) GO TO 401
      DO 2 I=1,M
      X(1)=W(I)**4.0
      X(3)=-2.0*W(I)**2.0
      X(2)=0.0
      X(5)=1.0
      X(4)=0.0
      CALL PMPY(Z,IDIMZ,Y,IDIMY,X,5)
      DO 3 J=1,IDIMZ
      Y(J)=Z(J)
    3 CONTINUE
      IDIMY=IDIMZ
    2 CONTINUE
C
C  CONVERSION OF INDEPENDENT VARIABLE FROM  W  TO  S
C
      DO 26 I=3,IDIMY,4
      Y(I)=-1.0*Y(I)
```

```
   26 CONTINUE
C
C     FIND THE ROOTS OF NUMERATOR SQUARED FUNCTION
C
      MNUM=IDIMY-1
      CALL POLRT (Y,COF,MNUM,ROOTR,ROOTI,IER)
C
C
C     SELECT THE RIGHT HALF PLANE ROOTS OF NUMERATOR SQUARED
C     FUNCTION
C
      L=1
      DO 20 I=1,MNUM
      IF(ROOTR(I).GT.0.0) GO TO 20
      RNUMR(L)=ROOTR(I)
      RNUMI(L)=ROOTI(I)
      L=L+1
   20 CONTINUE
      MN=MNUM/2
      LK=L
      LZ=L-1
      IF(LZ.EQ.MN) GO TO 33
      LL=1.0
      DO 35 I=1,MNUM
      LX=L+1
      LY=I+1
      IF(RNUMI(LL).EQ.ROOTI(I).OR.RNUMI(LL).EQ.(-1.0*ROOTI
     *(I))) GO TO 35
      IF(LL.EQ.1) GO TO 36
      LJ=LL-2
      IF(RNUMI(LJ).EQ.ROOTI(I).OR.RNUMI(LJ).EQ.(-1.0*ROOTI
     *(I))) GO TO 35
      IF(LL.EQ.3) GO TO 36
      LI=LL-4
      IF(RNUMI(LI).EQ.ROOTI(I).OR.RNUMI(LI).EQ.(-1.0*ROOTI
     *(I))) GO TO 35
   36 RNUMR(L)=ROOTR(I)
      RNUMI(L)=ROOTI(I)
      RNUMR(LX)=ROOTR(LY)
      RNUMI(LX)=ROOTI(LY)
      IF(LX.EQ.MN) GO TO 33
      LL=LL+2
   35 CONTINUE
   33 DO 22 I=1,MN
      R(I)=CMPLX(RNUMR(I),RNUMI(I))
   22 CONTINUE
      CALL MAKPOL (MN,R,C)
      MN1=MN+1
      C(MN1)=CMPLX(1.0,0.0)
      DO 31 I=1,MN1
      RC(I)=REAL(C(I))
   31 CONTINUE
      GO TO 5
  401 RC(1)=1.0
      MN1=1
      MN=0
      RNUMR(1)=0.0
      RNUMI(1)=0.0
C
C     CALCULATION OF DENOMINATOR POLYNOMIAL
C
    5 NDNUM=2*N
      IF(M.EQ.0.AND.K.EQ.N) GO TO 402
      CONST=1.0
      IF(M.EQ.0) GO TO 7
      DO 11 I=1,M
      CMULT=(W(I)**2.0-1.0)**2.0
      CONST=CMULT*CONST
   11 CONTINUE
    7 IF(K.EQ.0) GO TO 52
      KK=2*K
      KKK=KK+1
```

```
      CM(KKK)=CONST
      DO 12 I=1,KK
      CM(I)=0.0
   12 CONTINUE
      GO TO 53
   52 CM(1)=1.0
      KK=0.0
      KKK=1.0
   53 NK=N-K
      NKK1=NK+1
C
C   OBTAIN THE CHEBYSHEV POLYNOMIAL,SQUARE IT AND MULTIPLY
C   THE RESULT BY BUTTERWORTH SQUARED FUNCTION
C
      CALL CHEBV(CY1,NK)
      CALL PMPY (CY,NKK,CY1,NKK1,CY1,NKK1)
      CALL PMPY (CZ,IDIMCZ,CY,NKK,CM,KKK)
C
C   OBTAIN DENOMINATOR SQUARED FUNCTION
C
      CALL PADDM(Z,IDIMZ,Y,IDIMY,1.0,CZ,IDIMCZ)
C
C   CONVERSION OF INDEPENDENT VARIABLE FROM W TO S
C
    6 DO 17 I=3,IDIMZ,4
      Z(I)=-1.0*Z(I)
   17 CONTINUE
    8 NONUM=IDIMZ-1
C
C   FIND THE ROOTS OF DENOMINATOR SQUARED FUNCTION
C
      CALL POLRT (Z,COF,NONUM,ROOTOR,ROOTOI,IER)
      GO TO 403
  402 CALL ROOT(NONUM,ROOTOR,ROOTOI)
C
C   SELECT THE RIGHT  HALF PLANE ROOTS OF DENOMINATOR SQUARED
C   FUNCTION
C
  403 J=1
      DO 21 I=1,NONUM
      IF(ROOTOR(I).GT.0.0) GO TO 21
      RDNUMR(J)=ROOTOR(I)
      RDNUMI(J)=ROOTOI(I)
      J=J+1
   21 CONTINUE
      MD=NONUM/2
      DO 23 I=1,MD
      R1(I)=CMPLX(RDNUMR(I),RDNUMI(I))
   23 CONTINUE
      CALL MAKPOL (MD,R1,C1)
      MD1=MD+1
      C1(MD1)=CMPLX(1.0,0.0)
   47 FORMAT(2X,25F11.3//)
      DO 32 I=1,MD1
      RC1(I)=REAL(C1(I))
   32 CONTINUE
C
C   NORMALIZATION OF NUMERATOR POLYNOMIAL
C
      FACTOR=RC1(1)/RC(1)
      DO 100 I=1,MN1
      RC(I)=RC(I)*FACTOR
  100 CONTINUE
      WRITE(6,570)
C
C   OUTPUT SECTION
C
  570 FORMAT('1'////)
      WRITE(6,560)
  560 FORMAT(2X,' LOW-PASS PROTOTYPE(CONTINUOUS) FILTER'//)
      WRITE(6,571)
  571 FORMAT(2X,' ORDER OF FILTER '//)
```

```
      WRITE(6,800) N
  800 FORMAT(2X,I2//)
      WRITE(6,572)
  572 FORMAT(2X,' ORDER OF INSERTED ZEROS '//)
      WRITE(6,800) M
      WRITE(6,573)
  573 FORMAT(2X,' WEIGHTING FACTOR OF TBC FILTER '//)
      WRITE(6,800) K
      WRITE(6,574)
  574 FORMAT(2X,' LOCATIONS OF ZEROS '//)
      WRITE(6,801) (W(I),I=1,M)
  801 FORMAT(8F10.5//)
      WRITE(6,578)
  578 FORMAT(2X,' COEFFICIENTS OF NUMERATOR POLYNOMIAL
     *( DESCENDING ORDER ) '//)
      WRITE(6,47) (RC(I),I=1,MN1)
      WRITE(6,575)
  575 FORMAT(2X,' COEFFICIENTS OF DENOMINATOR POLYNOMIAL
     *( DESCENDING ORDER ) '//)
      WRITE(6,47) (RC1(I),I=1,MD1)
      WRITE(6,511)
  511 FORMAT(2X,'ROOTS OF NUMERATOR'//)
      WRITE(6,512)
  512 FORMAT(6X,'REAL PART',4X,'IMAGINARY PART '//)
      DO 520 I=1,MN
      WRITE(6,513) RNUMR(I),RNUMI(I)
  513 FORMAT(5X,F10.5,4X,F10.5/)
  520 CONTINUE
      WRITE(6,514)
  514 FORMAT(2X,'ROOTS OF DENOMINATOR'//)
      WRITE(6,512)
      DO 521 I=1,MD
      WRITE(6,513) RDNUMR(I),RDNUMI(I)
  521 CONTINUE
      WRITE(6,576)
  576 FORMAT(2X,' INITIAL VALUE OF FREQUENCY '//)
      WRITE(6,510) WBEGIN
  510 FORMAT(2X,F10.5//)
      WRITE(6,577)
  577 FORMAT(2X,' FINAL VALUE OF FREQUENCY '//)
      WRITE(6,510) WLAST
C
C   PLOTTING SECTION
C
      WRITE(6,590)
  590 FORMAT('1')
C
C   CALL PPLOT TO FIND FREQUENCY RESPONSE VALUES OF THE
C   FILTER
C
      CALL PPLOT(RC,RC1,WBEGIN,WDELTA,NPOINT,PX,PY,MD1,MN1)
      CALL PLOTP (PX,PY,NPOINT,0)
C
C   PREDISTORTION FOR BILINEAR TRANSFORMATION
C
      SCALE=TAN(T/2.)
      DO 777 LK=1,MN1
      EXP=LK-1
      RN(LK)=RC(LK)/(SCALE**EXP)
  777 CONTINUE
C
      DO 778 LK=1,MD1
      EXP=LK-1
      RD(LK)=RC1(LK)/(SCALE**EXP)
  778 CONTINUE
      MT=MN
      MK=MD
      CALL ZDMN (RN,RD,MK,MT,WBEGIN,WLAST,WDELTA,NPOINT,T)
      CALL XFORM(RC,RC1,1,XT,YT,MN,MD,1000.0,12.0)
      CALL PPLOT(XT,YT,500.0,10.0,100,AX,AY,MD1,MN1)
      CALL PLOTP(AX,AY,100,0)
      STOP
```

```
      END


      SUBROUTINE CHBSV (CY,NC)
C
C   SUBROUTINE TO FIND THE COEFFICIENTS OF CHEBYSHEV
C   POLYNOMIAL OF GIVEN ORDER
C   NC : ORDER OF CHEBYSHEV POLYNOMIAL
C   CY : CALCULATED COEFFICIENTS (DESCENDING ORDER)
C
      DIMENSION CX(50),CY(50),YY(50),ZZ(50),Z(50)
      NN=NC+1
      CX(1)=1.0
      IF(NC.EQ.0) GO TO 2
      CY(1)=0.0
      CY(2)=1.0
      IF(NC.EQ.1) GO TO 19
      YY(1)=0.0
      YY(2)=2.0
      DO 5 I=2,NC
      II=I+1
      I1=I-1
      CALL PMPY(Z,IDIMZ,YY,2,CY,I)
      IDIMZZ=IDIMZ
      DO 6 J=1,IDIMZ
      ZZ(J)=Z(J)
    6 CONTINUE
      CALL PSUB(Z,IDIMZ,ZZ,IDIMZZ,CX,I1)
C
C   UPDATE POLYNOMIALS
C
      DO 7 J=1,I
      CX(J)=CY(J)
    7 CONTINUE
      DO 8 J=1,IDIMZ
      CY(J)=Z(J)
    8 CONTINUE
    5 CONTINUE
   19 RETURN
    2 CY(1)=CX(1)
      GO TO 19
      END


      SUBROUTINE ZDMN(RC,RC1,N,M,F1,F2,WDELTA,NP,T)
C
C   SUBROUTINE TO FIND AND PLOT FREQUENCY RESPONSE OF DIGITAL
C   FILTERS
C   RC1 : COEFFICIENTS OF DENOMINATOR OF CONTINUOUS FUNCTION
C   RC COEFFICIENTS OF NUMERATOR OF CONTINUOUS FUNCTION
C   N : ORDER OF DENOMINATOR
C   M : ORDER OF NUMERATOR
C   F1 : INITIAL FREQUENCY
C   F2 : FINAL FREQUENCY
C   T : SAMPLING PERIOD
C
      DIMENSION BCC(25)
      DIMENSION RC(25),RC1(25),ZX(25),ZX1(25),XJ(25),YJ(25),
     *ROOTRD(25),ROOTRN(25),ROOTID(25),ROOTIN(25),CY(25),
     *XPAY(25),XPAYDA(25),
     *CX(25),YPAY(3),YPAYDA(3)
      DIMENSION A00(10),A11(10),A22(10),B11(10),B22(10)
      DIMENSION XAXIS(1500),YAXIS1(1500),YAXIS2(1500),
     *PH(1500),DEL(1500)
      DIMENSION XCOF(3),X2COF(3),COF(3),ROOTR(2),ROOTI(2)
     1,ROOT1(2),ROOT2(2)
      DIMENSION DUMKAY(100)
      DOUBLE PRECISION W,Y
      COMMON/RAW/W(100),Y(100),ICALL
      REAL MAGH,MAGNH,NUM1,NUM2
```

```
      COMPLEX CX,CY,XJ,YJ
      COMPLEX NUM,DENOM
      MJ=M+1
      NJ=N+1
      NMM=N-M
      CALL BZXFRM(RC,M,NMM,ZX,1)
      CALL BZXFRM(RC1,N,NMM,ZX1,0)
C
      SFACT=ZX(NJ)/ZX1(NJ)
      DO 90 I=1,NJ
      ZX1(I)=ZX1(I)*SFACT
   90 CONTINUE
C
      WRITE(6,60)
C
C COEFFICIENTS OF DENOMINATOR POLYNOMIAL
C
      WRITE(6,45)(ZX1(I),I=1,NJ)
      WRITE(6,60)
      CALL POLRT(ZX,COF,N,ROOTRN,ROOTIN,IER)
      CALL POLRT(ZX1,COF,N,ROOTRD,ROOTID,IER)
      CALL ODEVEN(N,MX)
      CALL FACTOR(ROOTRN,ROOTIN,N,MX,A00,A11,A22)
      CALL FACTOR(ROOTRD,ROOTID,N,MX,B22,B11,B00)
      F=F1
      DO 10 I=1,NP
      X=F*T
      MAGH=1.0
      PH3=0.0
      DO 6 J=1,MX
      A0=A00(J)
      A1=A11(J)
      A2=A22(J)
      B1=B11(J)
      B2=B22(J)
      NUM1=(A2+A0)*COS(X)+A1
      NUM2=(A0-A2)*SIN(X)
      NUM=CMPLX(NUM1,NUM2)
      DENOM1=(B2+1.0)*COS(X)+B1
      DENOM2=(1.-B2)*SIN(X)
      DENOM=CMPLX(DENOM1,DENOM2)
      MAGH=ABS(MAGH*CABS(NUM/DENOM))
      PH1=(ATAN(NUM2/NUM1))*57.29577951
      PH2=(ATAN(DENOM2/DENOM1))*57.29577951
      PH3=PH3+(PH1-PH2)
    6 CONTINUE
      MAGNH=20.*ALOG10(MAGH)
      PH(I)=PH3
      PK=PH(I)
      XAXIS(I)=F
      YAXIS1(I)=MAGH
      YAXIS2(I)=MAGNH
      W(I)=XAXIS(I)
      Y(I)=YAXIS1(I)
      F=F+WDELTA
   10 CONTINUE
      WRITE(6,60)
   60 FORMAT('1')
   45 FORMAT(2X,10F12.3///)
      WRITE(6,70)
   70 FORMAT(2X,' BILINEARLY TRANSFORMED LOW-PASS
     *(PROTOTYPE) DIGITAL FILTER'//)
      WRITE(6,61)
   61 FORMAT(2X,' COEFFICIENTS OF NUMERATOR  POLYNOMIAL '//)
      WRITE(6,45)(ZX(I),I=1,NJ)
      WRITE(6,62)
   62 FORMAT(2X,'COEFFICIENTS OF DENOMINATOR POLYNOMIAL '//)
      WRITE(6,45)(ZX1(I),I=1,NJ)
      WRITE(6,63)
   63 FORMAT(2X,' NUMBER OF CASCADED STAGES'//)
      WRITE(6,64) MX
   64 FORMAT(1X,I3)
```

```
      WRITE(6,81)
   81 FORMAT(2X,'COEFFICIENTS OF CASCADED STAGES'//)
      DO 82 I=1,MX
      WRITE(6,83)A00(I),A11(I),A22(I),B11(I),B22(I)
   83 FORMAT(2X,5F12.5//)
   82 CONTINUE
      WRITE(6,65)
   65 FORMAT(2X,'ROOTS OF NUMERATOR'//)
      DO 66 I=1,N
      WRITE(6,45) ROOTRN(I),ROOTIN(I)
   66 CONTINUE
      WRITE(6,67)
   67 FORMAT(2X,' ROOTS OF DENOMINATOR '//)
      DO 68 I=1,N
      WRITE(6,45) ROOTRD(I),ROOTID(I)
   68 CONTINUE
      WRITE(6,98)
   98 FORMAT(2X,' SAMPLING FREQUENCY '//)
      WRITE(6,99)T
   99 FORMAT(2X,F10.5//)
      N=NP
      WRITE(6,13)
   13 FORMAT('1')
      CALL PLOTP(XAXIS,YAXIS1,N,O)
      WRITE(6,51)
   51 FORMAT(//40X,' ABS. GAIN V.S. FREQUENCY')
      WRITE(6,16)
   16 FORMAT('1')
      CALL PLOTP(XAXIS,YAXIS2,N,O)
      WRITE (6,52)
   52 FORMAT(//40X,' GAIN (DB ) V.S. FREQUENCY ')
      WRITE(6,14)
   14 FORMAT('1')
      CALL PLOTP(XAXIS,PH,N,O)
      WRITE(6,53)
   53 FORMAT(//40X,' PHASE V.S. FREQUENCY ')
      RETURN
      END


      SUBROUTINE ODEVEN(N,M)
C
C     SUBROUTINE TO FIND THE NUMBER OF CASCADED STAGES
C     N: DEGREE OF NUMERATOR POLYNOMIAL
C     M: DEGREE OF DENOMINATOR POLYNOMIAL
C
      NN=(N/2)*2
      IF(N.NE.NN) GO TO 1
      M=N/2
      GO TO 2
    1 M=(N+1)/2
    2 RETURN
      END


      SUBROUTINE BZXFRM(X,M,NMM,ZX,IFACT)
      DIMENSION X(25),Y(25),ZX(25),XDUM(2)
     *,YDUM(2),Z1(25),Z2(25),X1(25),X2(25)
     *,XFACT(25),ZX1(25)
C     SUBROUTINE TO FIND THE BILINEAR Z-TRANSFORM OF A GIVEN
C     POLYNOMIAL
C     X: POLYNOMIAL
C     M: ORDER OF POLYNOMIAL
C     NMM=N-M ( THE DIFFERENCE BETWEEN THE ORDERS OF NUMERATOR
C     AND DENOMINATOR
C     ZX: RESULTANT POLYNOMIAL
C     IFACT= 1 FOR NUMERATOR
C     IFACT= 0 FOR DENOMINATOR
C
      NM=M+NMM+1
```

```
      M2=M+1
      DO 1 I=1,MM
      ZX(I)=0.0
    1 CONTINUE
      XDUM(1)=-1.0
      XDUM(2)=1.0
      YDUM(1)=1.0
      YDUM(2)=1.0
      IZX=2
      MM=0
      DO 2  II=1,M2
      CALL POLEXP(XDUM,2,MM,X1,I1)
      CALL POLEXP(YDUM,2,M,X2,I2)
      CALL PMPY(Z1,IZ,X1,I1,X2,I2)
      FACT=X(II)
      CALL PADDM(Z2,IZ2,ZX,IZX,FACT,Z1,IZ)
      DO 4 K=1,IZ2
      ZX(K)=Z2(K)
    4 CONTINUE
      MM=MM+1
      M=M-1
      IZX=IZ2
    2 CONTINUE
      IF(IFACT.EQ.0) GO TO 5
      CALL POLEXP(YDUM,2,NMM,XFACT,NXFACT)
      CALL PMPY(ZX1,MZ,XFACT,NXFACT,ZX,IZ2)
      DO 6 I=1,MZ
      ZX(I)=ZX1(I)
    6 CONTINUE
    5 M=M2-1
      RETURN
      END


      SUBROUTINE FACTOR(RRN,RIN,M,MX,A22,A11,A00)
      DIMENSION RRN(25),RIN(25),XR(25),XI(25),YR(25),YI(25),
     * C(25),C1(25),A00(25),A11(25),A22(25)
      COMPLEX C,C1
C
C  INITIALIZE COUNTERS
C
      IM=0
      IY=1
      K=0
      NI=0
      IX=1
      NXR=0
      NXI=0
      IF(M.NE.0) GO TO 18
      DO 17 I=1,MX
      A00(I)=1.0
      A11(I)=0.0
      A22(I)=0.0
   17 CONTINUE
      GO TO 20
   18 IF(M.NE.1) GO TO 16
      A00(1)=1.0
      A11(1)=-1.0*RRN(1)
      A22(1)=0.0
      K=2
      NDIF=MX
      NXK=1
      GO TO 10
   16 DO 1 I=1,100
      IX1=IX+1
      IF(IX.GT.M) GO TO 7
      IF(RRN(IX).EQ.RRN(IX1).AND.RIN(IX).EQ.
     *(-1.0*RIN(IX1))) GO TO 2
      IM=IM+1
      XR(IM)=-1.0*RRN(IX)
      IX=IX+1
```

```
      NXR=NXR+1
      GO TO 1
    2 IY1=IY+1
      YR(IY)=RRN(IX)
      YI(IY)=RIN(IX)
      YR(IY1)=RRN(IX1)
      YI(IY1)=RIN(IX1)
      IX=IX1+1
      NI=NI+2
      NXI=NXI+1
      IY=IY1+1
    1 CONTINUE
    7 NT=NXR+NXI
      IF(NT.EQ.MX) GO TO 3
      IF(MX.GT.NT) GO TO 8
      NDIF=NT-MX
      IC=1
      DO 4 I=1,NDIF
      IC1=IC+1
      A00(I)=1.0
      A11(I)=XR(IC)+XR(IC1)
      A22(I)=XR(IC)*XR(IC1)
      IC=IC+1
    4 CONTINUE
      NDIF1=NDIF+1
      NCOF=NDIF*2
      NLAST=MX-NXI
      DO 5 I=NDIF1,NLAST
      NCOF=NCOF+1
      A00(NDIF1)=1.0
      A11(NDIF1)=XR(NCOF)
      A22(NDIF1)=0.0
    5 CONTINUE
      NBEGIN=NLAST+1
      NCOMP=1
      DO 6 I=NBEGIN,MX
      NCOMP1=NCOMP+1
      D(1)=CMPLX(YR(NCOMP),YI(NCOMP))
      D(2)=CMPLX(YR(NCOMP1),YI(NCOMP1))
      CALL MAKPOL(2,D,D1)
      A00(NBEGIN)=1.0
      A11(NBEGIN)=REAL(D1(2))
      A22(NBEGIN)=REAL(D1(1))
      NCOMP=NCOMP1+1
    6 CONTINUE
      GO TO 20
    8 NDIF=MX-NT
      IF(NXR.EQ.0) GO TO 15
      DO 9 I=1,NXR
      A00(I)=1.0
      A11(I)=XR(I)
      A22(I)=0.0
      K=K+1
    9 CONTINUE
      IF(NXI.EQ.0) GO TO 10
   15 NXK=NXR
      DO 11 I=1,NXI
      NXK=NXK+1
      IK=I+1
      D(1)=CMPLX(YR(I),YI(I))
      D(2)=CMPLX(YR(IK),YI(IK))
      CALL MAKPOL (2,D,D1)
      A00(NXK)=1.0
      A11(NXK)=REAL(D1(2))
      A22(NXK)=REAL(D1(1))
      K=K+1
   11 CONTINUE
   10 DO 12 I=K,NDIF
      NXK=NXK+1
      A00(NXK)=1.0
      A11(NXK)=0.0
      A22(NXK)=0.0
```

```
 12 CONTINUE
    GO TO 20
  3 IX=1
    DO 13 I=1,NXR
    A00(I)=1.0
    A11(I)=XR(I)
    A22(I)=0.0
 13 CONTINUE
    NXX=NXR+1
    DO 14 I=NXX,MX
    IX1=IX+1
    D(1)=CMPLX(YR(IX),YI(IX))
    D(2)=CMPLX(YR(IX1),YI(IX1))
    CALL MAKPOL (2,D,D1)
    A00(NXX)=1.0
    A11(NXX)=REAL(D1(2))
    A22(NXX)=REAL(D1(1))
    IX=IX1+1
    NXX=NXX+1
 14 CONTINUE
 20 RETURN
    END


    SUBROUTINE POLEXP(XX,IDIMXX,M,YY,IDIMYY)
C
C   SUBROUTINE TO FIND THE POWERS OF GIVEN POLYNOMIAL
C   XX : POLYNOMIAL
C   IDIMXX : DIMENSION OF XX
C   M: POWER TO BE RAISED
C   YY:RESULTANT POLYNOMIAL
C   IDIMYY: DIMENSION OF RESULTANT POLYNOMIAL
C
    DIMENSION XX(25),YY(25),ZZ(25)
    IF(M.EQ.1) GO TO 4
    IDIMYY=1
    YY(1)=1.0
    IF(M.EQ.0) GO TO 3
    DO 1 I=1,M
    CALL PMPY (ZZ,IDIMZZ,YY,IDIMYY,XX,IDIMXX)
    DO 2 J=1,IDIMZZ
    YY(J)=ZZ(J)
  2 CONTINUE
    IDIMYY=IDIMZZ
  1 CONTINUE
    GO TO 3
  4 DO 5 I=1,IDIMXX
    YY(I)=XX(I)
  5 CONTINUE
    IDIMYY=IDIMXX
  3 RETURN
    END


    SUBROUTINE PPLOT(RO,RO1,WBEGIN,WDELTA,NPOINT,PX,PY,
   *MO1,MN1)
    DIMENSION RO(100),RO1(100)
    DIMENSION PX(100),PY(100)
    COMPLEX XNUM,DNUM
    W=WBEGIN
C
    DO 666 I=1,NPOINT
    W=W+WDELTA
C
C   INITIALIZATION OF REAL AND COMPLEX PARTS OF NUMERATOR
C   AND DENOMINATOR
C
    XNUMA=0.0
    XNUMB=0.0
    XNUMC=0.0
```

```
      XNUMD=0.0
C
      XDNUMA=0.0
      XDNUMB=0.0
      XDNUMC=0.0
      XDNUMD=0.0
C
C  INITIALIZATION OF COUNTERS
C
      MNUMO=1
      MNUM1=2
      MNUM2=3
      MNUM3=4
C
      MDNUMO=1
      MDNUM1=2
      MDNUM2=3
      MDNUM3=4
C
      X0=MNUMO-1
      X1=MNUM1-1
      X2=MNUM2-1
      X3=MNUM3-1
C
      XD0=MDNUMO-1
      XD1=MDNUM1-1
      XD2=MDNUM2-1
      XD3=MDNUM3-1
C
      DO 665 J=1,100
C
      IF(MNUM3.LE.MN1)  XNUMD=XNUMD+RC(MNUM3)*W**X3
      IF(MNUM2.LE.MN1)  XNUMC=XNUMC+RC(MNUM2)*W**X2
      IF(MNUM1.LE.MN1)  XNUMB=XNUMB+RC(MNUM1)*W**X1
      IF(MNUMO.LE.MN1)  XNUMA=XNUMA+RC(MNUMO)*W**X0
C
      IF(MDNUM3.LE.MD1)  XDNUMD=XDNUMD+RC1(MDNUM3)*W**XD3
      IF(MDNUM2.LE.MD1)  XDNUMC=XDNUMC+RC1(MDNUM2)*W**XD2
      IF(MDNUM1.LE.MD1)  XDNUMB=XDNUMB+RC1(MDNUM1)*W**XD1
      IF(MDNUMO.LE.MD1)  XDNUMA=XDNUMA+RC1(MDNUMO)*W**XD0
C
C  INCREMENT COUNTERS
C
      MNUMO=MNUMO+4
      MNUM1=MNUM1+4
      MNUM2=MNUM2+4
      MNUM3=MNUM3+4
C
      MDNUMO=MDNUMO+4
      MDNUM1=MDNUM1+4
      MDNUM2=MDNUM2+4
      MDNUM3=MDNUM3+4
C
      X0=X0+4.0
      X1=X1+4.0
      X2=X2+4.0
      X3=X3+4.0
C
      XD0=XD0+4.0
      XD1=XD1+4.0
      XD2=XD2+4.0
      XD3=XD3+4.0
C
      IF((MDNUMO.GT.MD1).AND.(MNUMO.GT.MN1)) GO TO 668
  665 CONTINUE
C
C  CALCULATE MAGNITUDE OF THE TRANSFER FUNCTION
C
  668 XNUM1=XNUMA-XNUMC
      XNUM2=XNUMB-XNUMD
      XNUM=CMPLX(XNUM1,XNUM2)
      DNUM1=XDNUMA-XDNUMC
```

```
      DNUM2=XDNUMB-XDNUMD
      DNUM=CMPLX(DNUM1,DNUM2)
      XMAGH=CABS(XNUM/DNUM)
      PX(I)=W
      PY(I)=20.0*ALOG10(XMAGH)
  666 CONTINUE
      RETURN
      END


      SUBROUTINE XFORM(X,Y,NTYPE,XT,YT,M,N,F1,F2)
C
C  SUBROUTINE FOR FREQUENCY TRANSFORMATIONS
C
C  NTYPE=0 ; NO TRANSFORMATIONS REQUIRED
C  NTYPE=1 ; LOW-PASS TO LOW-PASS TRANSFORMATION
C  NTYPE=2 ; LOW-PASS TO HIGH-PASS TRANSFORMATION
C  NTYPE=3 ; LOW-PASS TO BAND-PASS TRANSFORMATION
C  NTYPE=4 ; LOW-PASS TO BAND-STOP TRANSFORMATION
C  X : COEFFICIENTS OF NUMERATOR POLYNOMIAL
C  Y : COEFFICIENTS OF DENOMINATOR POLYNOMIAL
C  M : ORDER OF NUMERATOR POLYNOMIAL
C  N : ORDER OF DENOMINATOR POLYNOMIAL
C  XT,YT : TRANSFORMED NUMERATOR AND DENOMINATOR POLYNOMIALS
C
      DIMENSION X(25),Y(25),XT(25),YT(25)
      NM=N-M
      N1=N+1
      M1=M+1
      IF(NTYPE.NE.1) GO TO 11
      DO 1 I=1,M1
      XM=N1-I
      XT(I)=X(I)*(F1**XM)
    1 CONTINUE
      DO 2 I=1,N1
      XN=N1-I
      YT(I)=Y(I)*(F1**XN)
    2 CONTINUE
      GO TO 20
   11 IF(NTYPE.NE.2) GO TO 20
      XF=M
      YF=N
      FACT=(F1**YF)/(F1**XF)
      K=M1+1
      DO 3 I=1,M1
      XM=M1-I
      K=K-1
      XT(I)=(X(K)*(F1**XM))*FACT
    3 CONTINUE
      K=N1+1
      DO 4 I=1,N1
      XN=N1-I
      K=K-1
      YT(I)=Y(K)*(F1**XN)
    4 CONTINUE
   20 RETURN
      END


      SUBROUTINE ROOT(N,RR,RI)
C
C  SUBROUTINE TO FIND THE ROOTS OF THE EQUATION
C  X**N+1=0
C  N : ORDER OF POLYNOMIAL
C  RR :ARRAY CONTAINING REAL PARTS OF CALCULATED ROOTS
C  RI :ARRAY CONTAINING IMAGINARY PARTS OF CALCULATED ROOTS
C
```

```
      DIMENSION RR(25),RI(25)
      PI=3.141592
      XN=N
      TETA=PI/XN
      DO 1 I=1,N
      XI=I-1
      ARG=((2.0*XI)+1)*TETA
      RR(I)=SIN(ARG)
      RI(I)=COS(ARG)
    1 CONTINUE
      RETURN
      END
```

APPENDIX B

COMPUTER PROGRAM LISTING

```
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
C
C     SUBROUTINE TO FIND THE TIME DOMAIN RESPONSE
C     A DIGITAL FILTER
C
C     N : FILTER'S ORDER + 1
C     A : NUMERATOR COEFFICIENTS OF THE DIGITAL FILTER
C     B : DENOMINATOR COEFFICIENTS  ( WHERE FIRST COEFFICIENT
C     WILL BE NORMALIZED  AND WILL NOT BE ENTERED TO THE
C     PROGRAM
C
      P=1.
      N=24
      NPULSE=20
      WRITE(6,10)
   10 FORMAT('1')
      CALL INPULS
      CALL HMTRX
      CALL INPUT
      CALL CONVOL
      WRITE(6,10)
      CALL PLOT
      WRITE(6,10)
      STOP
      END


      SUBROUTINE HMTRX
C
C     SUBROUTINE TO FIND SYSTEMS TRANSFER MATRIX
C
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
      NN=N+1
      NM=NPULSE+1
      DO 1 I=1,NN
      DO 2 J=1,NN
      HM(I,J)=0.0
    2 CONTINUE
    1 CONTINUE
      DO 3 I=1,NN
      DO 4 J=1,NN
      IF(J.GT.I) GO TO 4
      L=I-J+1
      HM(I,L)=H(J)
    4 CONTINUE
    3 CONTINUE
      WRITE(6,20)
   20 FORMAT(2X,' HMATRIX '///)
   30 FORMAT(2X,26F5.3//)
      DO 5 I=1,NN
      WRITE(6,30) (HM(I,J),J=1,NN)
    5 CONTINUE
      RETURN
      END


      SUBROUTINE INPUT
C
C     SUBROUTINE TO FORM THE INPUT PULSE SEQUENCE
C
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
      NN=N+1
```

```
      NM=NPULSE+1
      DO 1 I=1,NPULSE
      X(I)=P
    1 CONTINUE
      DO 2 I=NM,NN
      X(I)=0.0
    2 CONTINUE
      WRITE(6,10)
   10 FORMAT(2X,' INPUT VECTOR '//)
      WRITE(6,20) (X(I),I=1,NN)
   20 FORMAT(2X,26F5.3//)
      RETURN
      END


      SUBROUTINE CONVOL
C
C   SUBROUTINE TO  PERFORM CONVOLUTION SUMMATION
C
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
      NN=N+1
      DO 1 I=1,NN
      V(I)=0.0
      DO 2 J=1,NN
      V(I)=V(I)+HM(I,J)*X(J)
    2 CONTINUE
    1 CONTINUE
      WRITE(6,10)
   10 FORMAT(2X,' RESULT VECTOR  '//)
      WRITE(6,20) (V(I),I=1,NN)
   20 FORMAT(2X,26F5.3//)
      RETURN
      END


      SUBROUTINE PLOT
C
C   SUBROUTINE TO PLOT THE OUTPUT
C
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
      DIMENSION T(250),Y(250),Z(250)
      NN=N+1
      NDELTA=10
      NPOINT=(NPULSE+3)*NDELTA
      DO 1 I=1,NPOINT
      XI=I
      T(I)=XI
      Y(I)=0.0
      Z(I)=0.0
    1 CONTINUE
      K=0
      DO 2 I=1,NPOINT,NDELTA
      IF(K.GT.NN) GO TO 5
      K=K+1
      DO 3 J=1,4
      L=I+J-1
      Y(L)=V(K)
    3 CONTINUE
    2 CONTINUE
    5 CALL PLOTP(T,Y,NPOINT,1)
C
C   PLOT INPUT PULSES
C
      K=0
      NM=NPULSE-1
      DO 10 I=1,NPOINT,NDELTA
      IF(K.GT.NM) GO TO 7
      K=K+1
      DO 8 J=1,4
      L=I+J-1
```

```
      Z(L)=X(K)
    8 CONTINUE
   10 CONTINUE
    7 CALL PLOTP(T,Z,NPOINT,3)
      RETURN
      END


      SUBROUTINE IMPULS
C
C   SUBROUTINE TO FIND IMPULSE RESPONSE OF THE FILTER.
C   ( SOLVING DIFFERENCE EQUATIONS )
C
      COMMON H(100),HM(100,100),X(100),V(100),N,P,NPULSE
      DIMENSION A(100),B(100)
      DIMENSION DUMMY(100)
      DIMENSION C(100)
      READ(5,30) M
      NN=N+1
      NNN=NN+20
C
C   INITIALIZATION OF VECTORS
C
      DO 35 I=1,NNN
      C(I)=0.
      H(I)=0.
      A(I)=0.
      B(I)=0.
   35 CONTINUE
      C(20)=1.
C
C   SHIFT ORIGIN TO 20
C
      NORDER=M+20
      READ(5,20)(A(I),I=20,NORDER)
      READ(5,20)(B(I),I=20,NORDER)
   30 FORMAT(I2)
   20 FORMAT(8F10.5)
      WRITE(6,1) M
    1 FORMAT(2X,' ORDER OF THE FILTER IS :',I2///)
      WRITE(6,2)
    2 FORMAT(2X,'NUMERATOR COEFFICIENTS '///)
      WRITE(6,3)(A(I),I=20,NORDER)
      WRITE(6,4)
    4 FORMAT(2X,'DENOMINATOR COEFFICIENTS '///)
      WRITE(6,3)(B(I),I=20,NORDER)
      DO 40 II=20,NNN
      FACT1=0.
      NK=II+1
      DO 50 IL=20,NORDER
      NK=NK-1
      FACT1=FACT1+A(IL)*C(NK)
   50 CONTINUE
      FACT2=0.
      NK=II
      DO 60 IL=21,NORDER
      NK=NK-1
      FACT2=FACT2+B(IL)*H(NK)
   60 CONTINUE
      H(II)=FACT1-FACT2
   40 CONTINUE
      WRITE(6,21)
   21 FORMAT(2X,'WEIGHTING SEQUENCE'///)
      WRITE(6,10)(H(I),I=20,NNN)
   10 FORMAT(2X,12F10.5//)
    3 FORMAT(2X,12F10.3)
      KKK=1
      DO 55 KK=20,NORDER
      A(KKK)=A(KK)
      B(KKK)=B(KK)
      KKK=KKK+1
```

```
 55 CONTINUE
    KK=1
    DO 65 I=20,NNN
    DUMMY(KK)=H(I)
    KK=KK+1
 65 CONTINUE
    DO 75 I=1,NN
    H(I)=DUMMY(I)
 75 CONTINUE
    XDUM=B(3)
    B(3)=B(1)
    B(1)=XDUM
    READ(5,100) F1,F2
    READ(5,100) T
100 FORMAT(3F10.5)
    NP=100
    XNP=100.
    FDELTA=(F2-F1)/XNP
    CALL ZDMN(A,B,M,M,F1,F2,FDELTA,NP,T)
    RETURN
    END
```

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